The Generation of Antigravity

by

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1. Introduction

The concept of antigravity refers to a force acting in a direction opposite to that of normal gravitation. The latter causes the earth to attract all objects towards its center, while objects under the influence of antigravity are repelled by the earth and either move upwards or experience a reduction in weight.

It has often been conjectured that UFOs might use an antigravitational drive when manoeuvring near the earth's surface without any visible means of propulsion. One indication supporting this view is the downward pressure sometimes exerted on objects underneath low-flying UFOs. Another is the appearance of strong electromagnetic fields in their vicinity. As will be seen, such fields of necessity accompany the generation of antigravity.

Apart from its possible utilization by UFOs, antigravity would find many useful applications here on earth if it could be produced in sufficient strength. The aim of the present study is to outline a method for generating antigravity and to describe the equipment needed for this. A word of caution at the very outset: The reader should not expect too much, for the effect turns out to be very small indeed if produced by present-day technology.

The calculations to follow are based on a 4-dimensional version of B. Heim's 6-dimensional unified field theory (Heim, 1989, 1984). Being 4-dimensional the theory to be presented can only be approximately correct. Previous publications on antigravity and related topics include articles by B. Heim (1959) and I. von Ludwiger (1976, 1979, 1983). Reports on observed gravitational effects due to UFOs are described by A. Schneider (1976, 1981).

The theory starts out from a set of relations very similar to Maxwell's equations, describing the interrelations between electric, magnetic, and gravitational fields, including the mesofield. The mesofield only acts on moving masses and bears about the same relation to the gravitational field as the magnetic field does to the electric field.

Two coupled equations of the set describe the phenomenon of anti–gravity. In Section 2 the first of these is solved separately because the resulting solution is simple and tractable, allowing modifications to be made with relative ease. The solution of both coupled equations is de–rived in Section 3. Section 4 contains a brief discussion of the anti–gravitational field and of the electromagnetic fields accompanying anti–gravity. In order to keep mathematics in the main text to a minimum full mathematical details are presented in Appendices A and B.

The antigravitational force turns out to be of measurable proportions only if the mass of the entire earth is utilized for repelling the device that generates the antigravity field. For this reason the theory to follow will be applied from the outset to methods of flight propulsion.

2. Dipole Solution of the First Equation

2.1. *The Basic Equation*

The equation to be solved in this section is

$$\nabla \times \mathbf{\Gamma} = -b\frac{\partial \mathbf{B}}{\partial t} , \tag{1}$$

where

$\mathbf{\Gamma}$ = gravitational field

$\mathbf{B}$ = magnetic field

b = coupling constant.

The symbols $\nabla\times$ and $\partial/\partial t$ denote the mathematical operators curl and partial time derivative, respectively.

Reading from right to left, Eq. (1) states that a space– and time–dependent magnetic field **B** induces a space– and time–dependent gravitational field Γ (gamma). As is well known, it also induces an electric field, but the electric field has been omitted from Eq. (1) be–cause it acts on electrical charges only. In the absence of charges it is legitimate to drop the electric field from the equations.

The gravitational field Γ acts on masses such that the product $m\mathbf{\Gamma}$ represents a real force if m denotes a mass. A force always acts in some direction, implying that $\mathbf{\Gamma}$ is a directed quantity. This fact is ex–pressed by the bold printing of $\mathbf{\Gamma}$. The same is true with regard to the magnetic field **B**.

In Eq. (1) the direction of **Γ** is seen to depend on that of **B**. Since the latter is to be produced artificially, care must be taken to ensure a direction of **B** inducing a repulsive gravitational field **Γ**, and not an attractive one. The fact that this is possible demonstrates the great advantage of **Γ** over ordinary gravity, whose direction is always attractive and cannot be manipulated in any way whatsoever. Gravitational and antigravitational forces merely differ in the plus– or minus–signs in front of them. For this reason the term "gravitational" in this report will often be used collectively to denote both types of forces.

Of essential importance with regard to the expected strength of antigravity is the magnitude of b in Eq. (1). Its value is given by

$$b = \sqrt{\frac{\varepsilon_0}{\alpha}} = 8.625 \times 10^{-11} \text{ coul/kg} \ , \qquad (2)$$

ε_0 = vacuum permittivity ($\varepsilon_0 = 8.854 \times 10^{-12}$ farad/m)
α = permittivity of space to gravity *) ($\alpha = 1.19 \times 10^{9}\ s^2 kg/m^3$) .

b is a coupling constant between the magnetic field **B** and the gravitational field Γ. Obviously, it is extremely small, so that even a strong **B** will induce a vanishingly small gravitational field. However, this is no cause for concern, because the same occurs in ordinary gravity, where a mass m creates an exceedingly weak gravitational field around itself, to which it is coupled by the small gravitational constant γ (gamma) ($\gamma = 6.67 \times 10^{-11}\ m^3/s^2kg$). γ is seen to be somewhat smaller than b. Nevertheless, when the weak gravitational field of m interacts with the huge mass of the earth, the result is an attractive force – the weight of m – which is not small by any means. Evidently, the enormous mass of the earth more than compensates for the weak gravitational field of m. By analogy, it may be argued that the weak Γ–field of Eq. (1), interacting with the earth, should result in a relatively strong antigravitational force roughly equal in magnitude to that of ordinary gravity. Unfortunately, this hypothesis is not supported by the facts for reasons that will become clear later on.

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*) This name was suggested by W.K. Allan

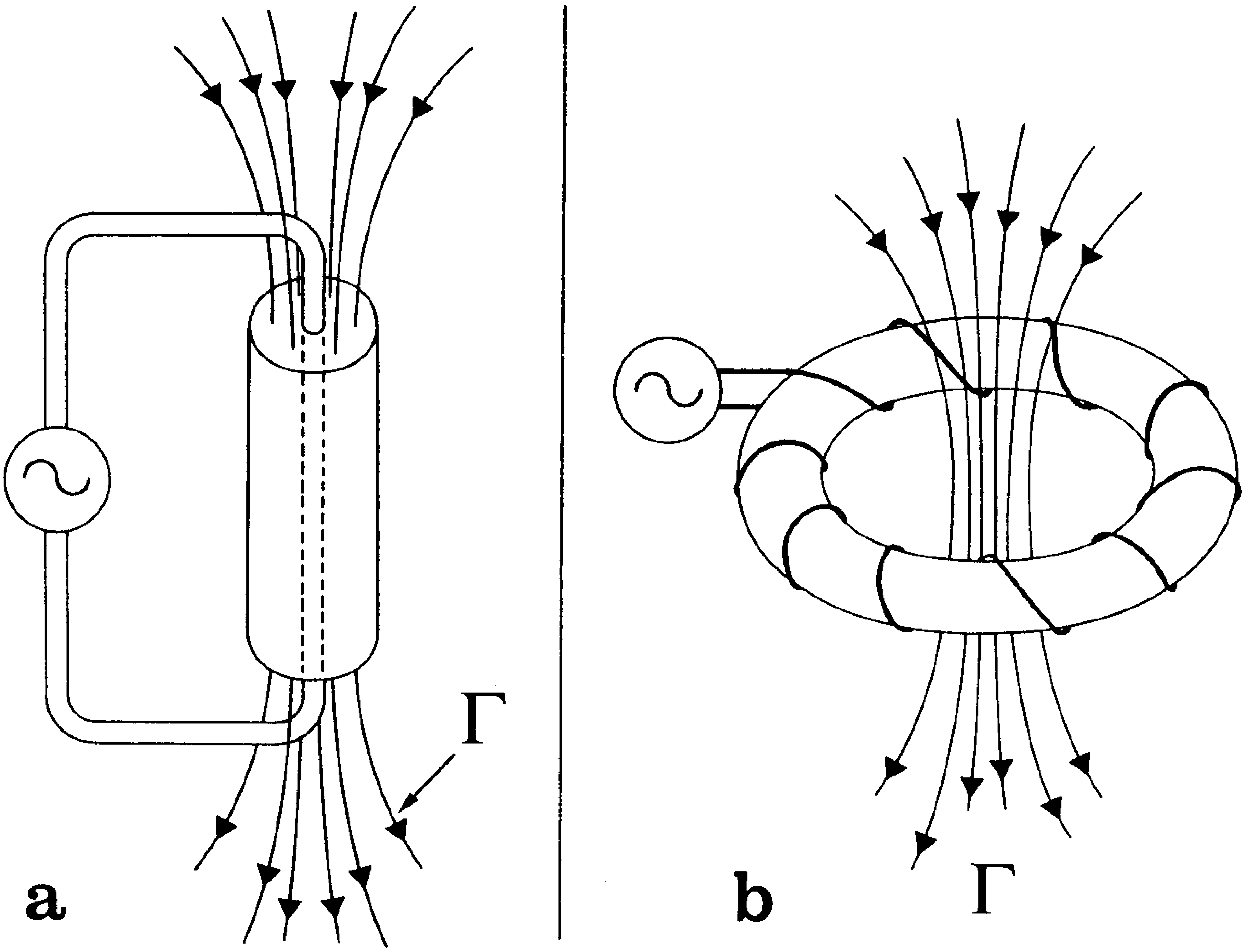

Fig. 1. Two possible configurations for the generation of an antigravitational field **Γ**.

2.2. *Equipment Needed for the Production of Antigravity*

Equation (1) can be solved after deciding on a magnetic field **B** suitable for generating antigravity. **B** is produced by an alternating current, since it has to be time–dependent. The field lines of **B** are known to be perpendicular to the current, while the field lines of **Γ** are perpendicular to those of **B**. Two possible arrangements meeting all necessary requirements are shown schematically in Fig. 1.

In Fig. 1a a rectangular current loop composed of many wires is connected to an alternating current source. The wires pass through a cylinder made of highly permeable material, such as iron. The current in–

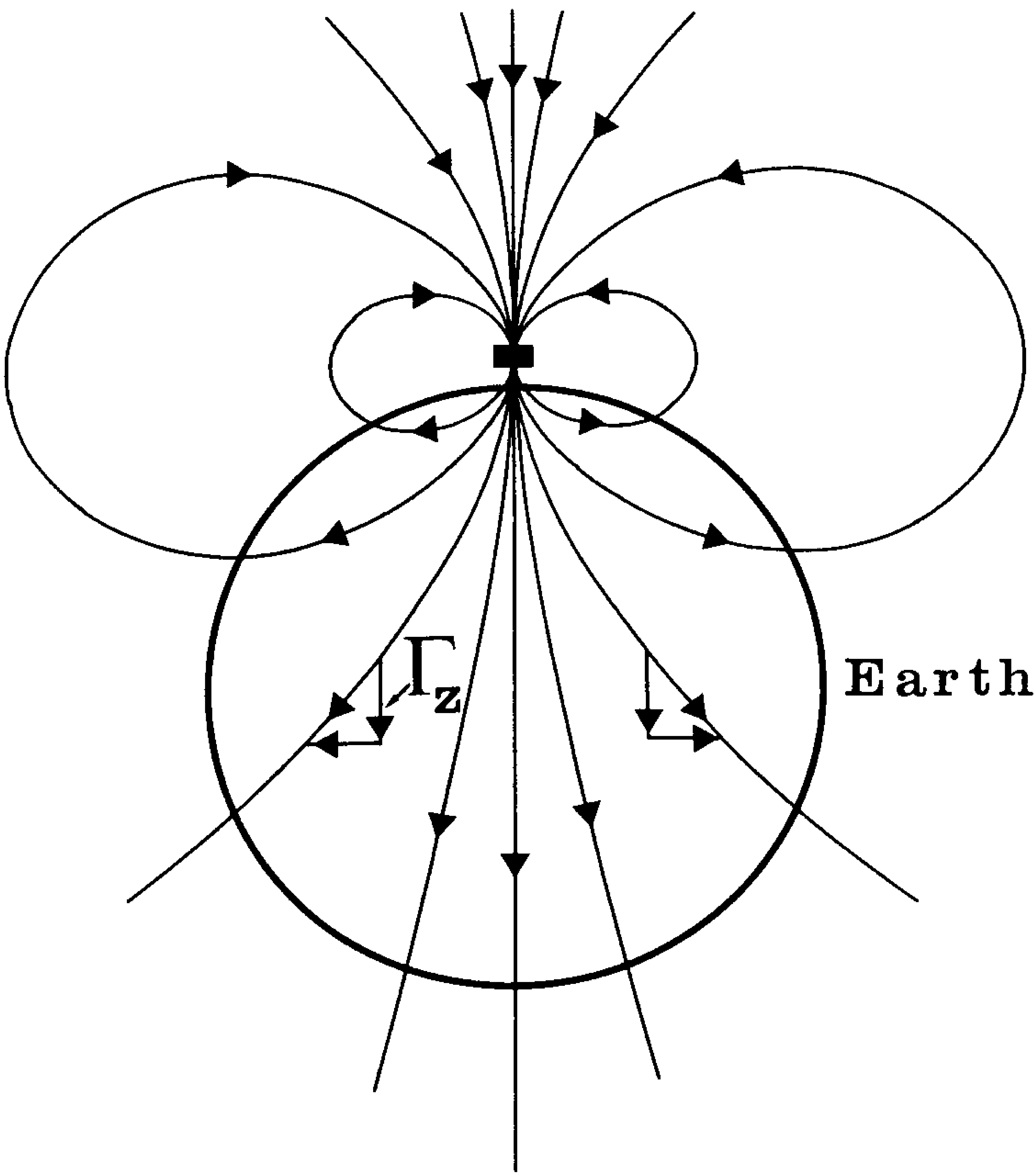

Fig. 2. Antigravitational field lines relative to the earth. Γ_z = vertical component of the antigravitational field.

duces a strong magnetic field **B**, whose field lines encircle the vertical portions of the loops in a plane perpendicular to them. The permeable material serves to increase the field strength of **B**. A strong **B** is needed because the antigravitational force is directly proportional to it. The curved field lines, belonging to Γ, in turn are induced by **B**. The gravitational force acts on any mass in the direction of the arrows tangentially to the field lines.

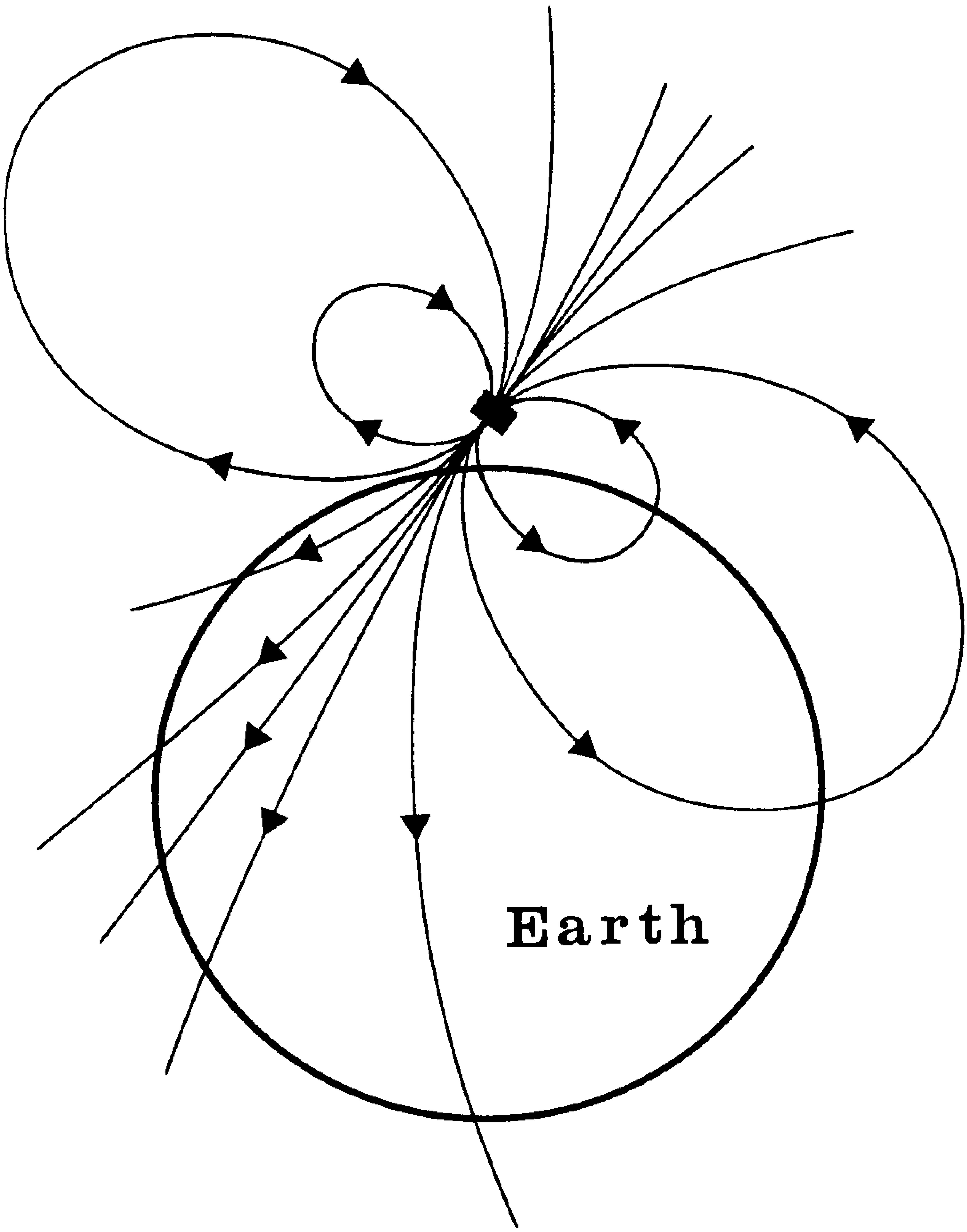

Fig. 3. A tilted magnet and its antigravitational field relative to the earth.

A second arrangement is shown in Fig 1b, where many windings surround a toroidal core of highly permeable material *). The magnetic field lines are circles inside the torus, while the field lines of Γ at some distance from the magnet are the same as in Fig. 1a. In the configuration of Fig. 1a end effects may cause a slight modification of the gravitational field relative to the field of Fig. 1b.

Calculated gravitational field lines based on the solution of Eq. (1) are shown in Fig. 2 in relation to the earth. They penetrate into the earth's

*) The author is indebted to A. Müller of MUFON–CES for suggesting the use of a toroidal magnet. See also v. Ludwiger (1979).

interior, acting on its mass at every point in the direction of the arrows. A visual inspection of Fig. 2 leads to the conclusion that the field pushes the earth away from the magnet in a downward direction. Since action equals reaction, the earth pushes the magnet with the same force in the upward direction. This is the desired antigravitational force acting on the magnet and on the entire construction attached to it.

Only the vertical component of the field, denoted by Γ_z in Fig. 2, contributes to the upward thrust. The horizontal components, pointing in opposite directions, cancel each other due to symmetry. This symmetry is disturbed if the magnet is tilted as in Fig. 3. The average force exerted on the earth by the arrows in the figure is downward and mainly to the left. In turn, the earth pushes the magnet up and to the right.

2.3. *The Vertical Field Component in Dipole Approximation*

An exact solution of Eq. (1) for the gravitational field $\boldsymbol{\Gamma}$ induced by the magnetic fields of Figs. 1a or 1b may be derived in the form of an infinite series of terms. If the height of the craft above the earth's surface is large compared to the dimensions of the magnet, which usually is the case, the first two terms of the series express the solution with sufficient accuracy for our purpose. The gravitational field resulting from this approximation is known as a dipole field.

The vertical component of the gravitational dipole field is given by the expression

$$\Gamma_z = -\frac{b\mu_0}{16\pi^2}\frac{3\cos^2\theta - 1}{r^3} V \frac{\mu}{\mu_0}\frac{dI}{dt} , \tag{3}$$

where r and θ are the coordinates of the point at which Γ_z is evaluated, as shown in Fig. 4, V is the volume of the magnet, μ/μ_0 is the relative permeability of its iron core, μ_0 is the permeability of free space ($\mu_0 = 1.257 \times 10^{-7}$ henry/m), I is the current producing the magnetic field, and dI/dt is the time derivative of I. Note the $1/r^3$–dependence of Γ_z. Ordinary gravity depends on $1/r^2$, i.e. it diminishes more slowly and remains stronger throughout the earth's volume. This leads us to suspect that antigravity may be a weaker force than normal gravity.

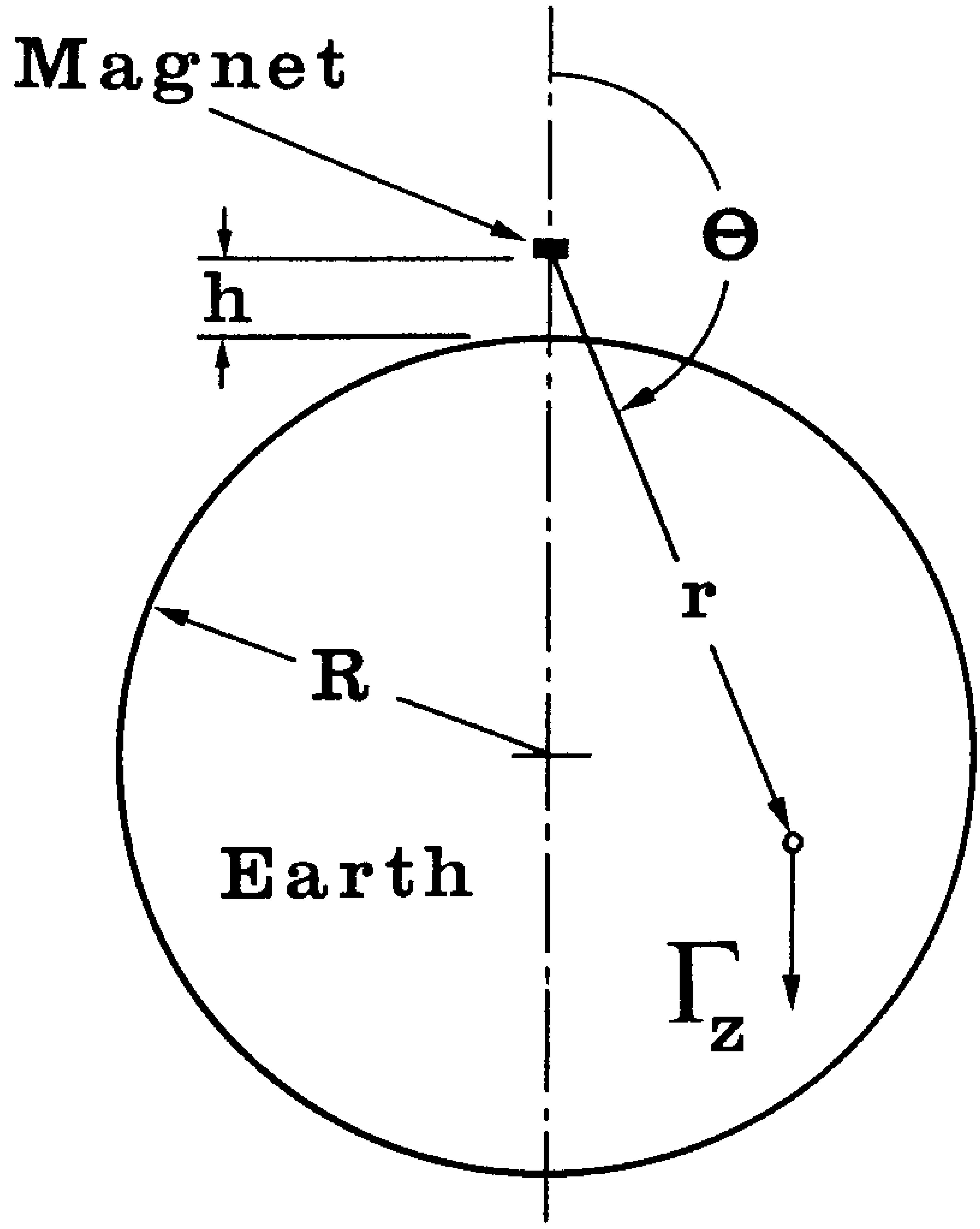

Fig. 4. Coordinate system for evaluating the vertical component of the gravitational dipole field, Γ_z.

2.4. *The Gravitational Force*

The last step in the calculation consists in evaluating the actual force acting on the magnet. To this end Γ_z of Eq. (3) is multiplied by the average density of the earth, ρ_m (rho), ($\rho_m = 5'500\ \mathrm{kg/m^3}$) and integrated over the earth's volume (an integration is a mathematical summation over many small elements). The force F resulting from this is

$$F = -3.16 \times 10^{-14} \frac{R^3}{(R + h)^3} V \frac{\mu}{\mu_0} \frac{dI}{dt} , \qquad (4)$$

where R is the earth's radius ($R = 6.378 \times 10^6$ m), and h is the height of the magnet above the earth's surface (cf. Fig. 4).

The main result of integration is the quotient $R^3/(R+h)^3$ in Eq. (4). Its magnitude is nearly equal to unity, because in general h is much smaller than R. When calculating the normal weight of an object a similar factor appears in the result, but there the denominator equals $(R+h)^2$ instead of $(R+h)^3$. Thus, if R is measured in meters, the quotient in Eq. (4) is about 6 million times smaller than the corresponding factor in ordinary gravity. Equation (4) confirms our suspicion that artificial antigravity is a much weaker force than gravity. Clearly, a fairly strong antigravitational force F can be attained only if the remaining quantities in Eq. (4) are made as large as reasonably possible.

2.5. *Elimination of the Time Dependence*

There now arises a new problem. The current I must be alternating. The simplest alternating current is sinusoidal and corresponds to the expression,

$$\begin{aligned} I &= I_0 \sin \omega t \\ \omega &= 2\pi f \ . \end{aligned} \tag{5}$$

Here I_0 is the maximum current, t is the time, ω (omega) is the angular frequency, and f is the frequency of oscillations. As an example, if $f = 100$ Hz (1 Hz (Hertz) = 1 oscillation per second) then $\omega = 200\,\pi = 628$ radians/second. According to Eq. (4) the force F is proportional to dI/dt which, with I given by Eq. (5), is

$$\frac{dI}{dt} = I_0\, \omega \cos \omega t \ . \tag{6}$$

When this is substituted into Eq. (4) the extra ω in Eq. (6) increases F substantially. One certainly should be careful not to lose ω again in subsequent mathematical operations. In our example f is equal to 100 Hz, but frequencies in the megahertz (MHz) range can easily be attained (ignoring self–inductance). This will result in very large values for ω, leading to an enormous increase in the antigravitational force F.

Unfortunately, this incease is only an illusion, because $\cos \omega t$ in Eq. (6) alternates between positive and negative values as time progresses, and so does F. If a negative value refers to antigravity, then a positive F is a force in the opposite direction, i.e. gravitational or attractive. As a result, F constantly alternates between an attractive and a repulsive force, whose average value is exactly zero. No sustained flight is possible under those circumstances.

A physical argument shows that no reasonable current can ever lead to acceptable flight conditions: The upward velocity which F is supposed to impart to the craft is the difference between two velocities: The first is the integral of F/m over time (m = mass of the craft), and the second is the velocity of free fall, gt (g = acceleration due to gravity = 10 m/s^2, t = time in seconds). If the net velocity is to be upwards, then the integral over F/m must be greater than gt. As the integral over F is proportional to I, this implies that the current should at least be proportional to t. But such a requirement is a physical impossibility, for time increases indefinitely, whereas a current may rise for a while, but eventually it has to come down again if no damage is to be done to the apparatus.

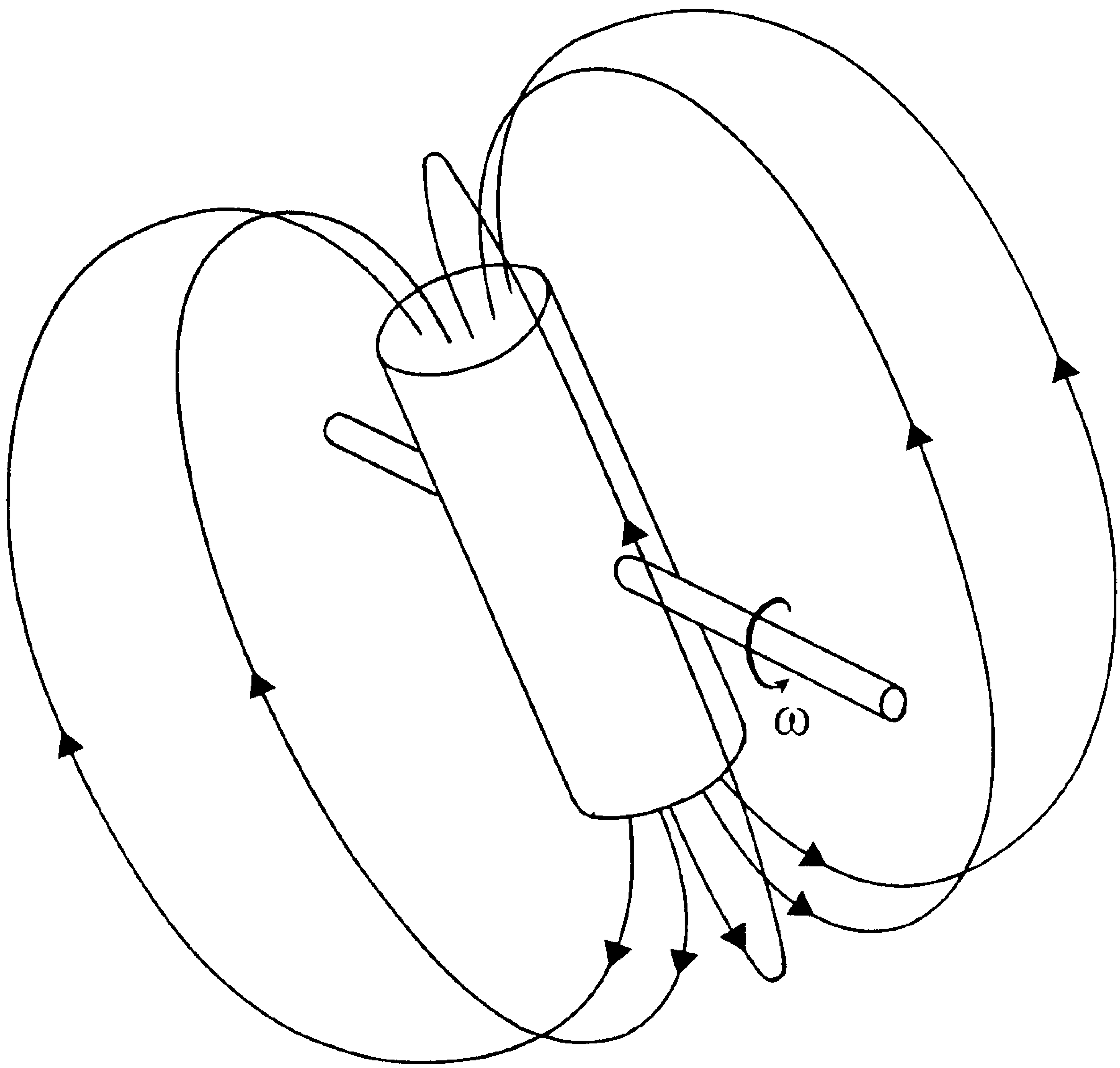

Fig. 5. Rotating magnet with its gravitational field. ω = angular frequency of rotation.

Evidently, it is impossible in principle to support sustained flight with the simple arrangement discussed so far. Brief flights of the craft may be possible during the antigravitational phase of F, followed by a hard landing as F turns gravitational, much to the passengers' discomfort. Moreover, the important factor ω in Eq. (6) is lost again when integrating over F.

Only one solution to the problem has come to mind so far, but hopefully there are others, and perhaps more elegant ones. The solution is to let the magnet rotate about a horizontal axis, together with its gravitational field, as shown in Fig. 5 for the cylinder of Fig. 1a. The rotational frequency must be the same as the current frequency, with the magnet rotating as $\cos \omega t$, i.e. 90° ahead of the sine–current.

As a result of rotation, an additional factor $\cos\omega t$ appears in Eqs. (3) and (4). Together with an $\omega\cos\omega t$ already contributed by dI/dt (cf. Eq. (6)) both the field and the force F now become proportional to $\cos^2\omega t$,

$$\Gamma_z,\ F \approx \omega\cos^2\omega t \quad . \tag{7}$$

The square of a number never changes sign, making F always anti–gravitational. When this F is integrated over time, ω is not lost, and in addition the integral contains a term proportional to t. This is precisely the time dependence needed for overcoming the velocity of free fall.

A second term resulting from the integration is proportional to $(1/\omega)\sin 2\omega t$. This term oscillates with twice the current frequency, but its effect is suppressed by the factor $1/\omega$. The angular frequency ω is restricted to relatively low values, because it has to match the rotational frequency of the magnet. As a result, the relatively slow oscillations may give rise to an uncomfortable vibration of the whole craft.

Another undesirable effect produced by the rotating magnet is a gyroscopic force affecting the craft's manoeuvrability by opposing any change in its direction of flight.

Fortunately, both problems can be solved by a single trick: Instead of using just a single magnet one uses two of them, rotating about the same axis but in opposite directions, as shown in Fig. 6. This immediately eliminates the gyroscope effect. Both magnets rotate with equal angular frequency ω, which must be the same as the current frequency. The second magnet is required to have a definite phase relation with respect to the first: Its axis must be horizontal when the axis of the

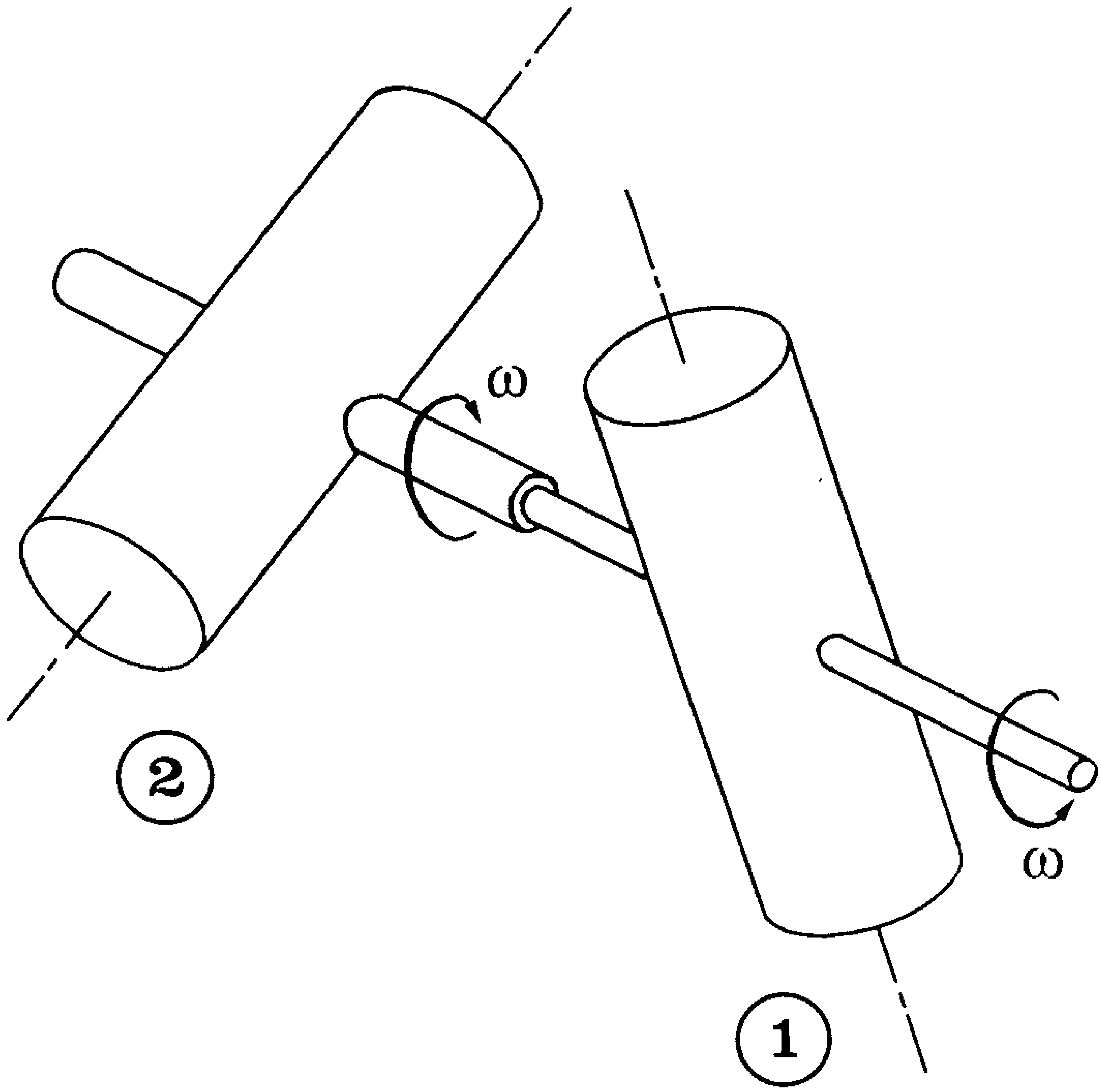

Fig. 6. Two counterrotating magnets.

first magnet is vertical and vertical when the first magnet is horizontal. Furthermore, a cosine current has to flow through the second magnet. As a result of counterrotation, an $\omega\cos^2\omega t$–term is introduced into the field of the first magnet as before, and an $\omega\sin^2\omega t$–term appears in the field of the second. When the two fields are added together the resulting force is proportional to

$$F \approx \omega(\sin^2\omega t + \cos^2\omega t) = \omega\ , \tag{8}$$

because the sum of the squares in Eq. (8) is always equal to unity.

The total antigravitational force now becomes,

$$F = -3.16 \times 10^{-14} \frac{R^3}{(R + h)^3} V \frac{\mu}{\mu_0} I_0 \omega\ . \tag{9}$$

This equation represents the ideal situation, for now F is antigravitational due to the minus-sign in front and entirely independent of time. Being constant, F is ideally suited for counteracting the constant weight of the craft. V in this and all subsequent formulas is the volume of only *one* of the two counterrotating magnets.

3. The Wave Equation

The solution of Eq. (1), represented by Eq. (9), turns out to be relatively simple. It enables one to investigate a number of schemes for eliminating the time dependence without too much effort. However, it is not yet the correct description of antigravity. As will be shown in Section 3.2, it is valid for very low frequencies only. Alternatively, it represents a much more general solution in the limiting case in which the velocity of light becomes infinite. It is this more general solution which leads to the correct expression for the antigravitational force.

It will be recalled that mention was made in the Introduction of *two* equations describing the phenomenon of antigravity. Only one of these has been solved so far. It expresses the fact that a time-dependent magnetic field induces time-dependent gravitational and electric fields. The second equation, which has not yet been solved, states that time-dependent gravitational and electric fields in turn induce a time-dependent magnetic field. The two equations are coupled, since the same fields appear in both. It is, therefore, not permissible to solve just one of them and to ignore the other.

The electric field induced in the first equation and the magnetic field induced in the second indicate that antigravitation is always accompanied by electromagnetic fields. The electric field turns out to be very strong and should result in strong electromagnetic disturbances near the rotating magnets.

The procedure for solving the two equations simultaneously again starts out by omitting the electric field for the same reason as before. Next, the magnetic field is eliminated, leaving a single relation involving the gravitational field alone. It is this equation which really must be solved and it differs substantially from Eq. (1). It turns out to be a wave equation, indicating that the gravitational field propagates away from the magnet in the form of a wave traveling with the speed of light.

The wave nature of the field has a profound effect on the gravitational force, as illustrated schematically in Fig. 7. The arrows along the vertical in Fig. 7 represent the vertical field component. The dotted line in

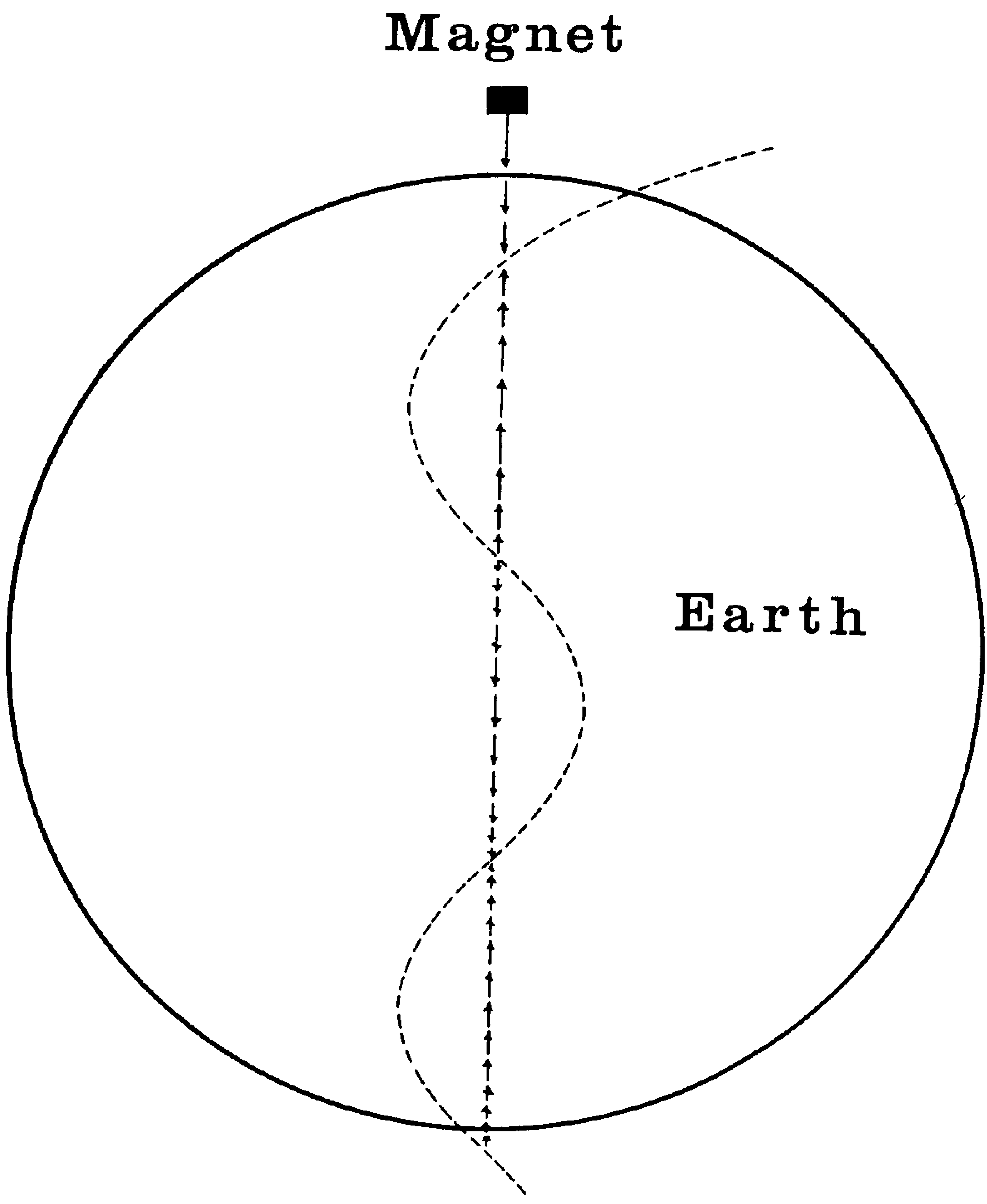

Fig. 7. Schematic view showing the wave nature of the gravitational field.

the form of a wave is merely intended as an optical aid, the actual wave being longitudinal (the wavelength in Fig. 7 is arbitrary). The whole wave pattern flows through the earth with the speed of light. Figure 7 represents a snapshot taken at the instant the antigravitational field (arrows pointing down) has a maximum at the position of the magnet. This fact is indicated by the extra long arrow underneath the magnet and by the large $1/r^3$–rise in amplitude of the dotted curve. At points farther away from the magnet the arrows become shorter because the field grows weaker due to its $1/r^3$–dependence, and because its wave nature reduces it still further. For the sake of clarity, the $1/r^3$–decrease of the dotted wave is neglected after the initial maximum. At the first node, where the dotted wave crosses the vertical axis, the field is zero. Thereafter it changes sign, becoming gravitational and hence attractive (arrows pointing up). Below the node the field is small at first, then grows larger and finally goes to zero again at the second node. This process is repeated throughout the earth and beyond. On the average, the field becomes weaker and the arrows grow shorter with increasing distance from the magnet due to the inverse–r dependence. The decrease is too rapid for the arrows in Fig. 7 to be drawn to scale.

A second snapshot, taken a fraction of a second later, would reveal a downward shift of the pattern due to its propagation with the speed of light, but again gravitational and antigravitational zones would be present in the earth's interior.

Evidently, attractive and repulsive regions are always present simultaneously, and this is bound to reduce the antigravitational force relative to its value in Eq. (9). In that equation an antigravitational field was assumed to exist at *every* point inside the earth. The wave equation leads to an approximately similar situation only when the wavelength λ (lambda) is equal to or larger than twice the earth's diameter. In that case the first node lies on or below the earth's circumference on the bottom of the circle in Fig. 7. For very short wavelengths, on the other hand, gravitational and antigravitational zones follow each other at such short intervals that the r–dependence is not effective enough for substantially changing the field strength from one zone to the next. As a consequence, the two opposing forces in adjacent zones are almost equally large and nearly cancel each other. The conclusion is that the antigravitational effect becomes weaker the shorter the wavelength or the higher the frequency.

3.1. *Solution of the Wave Equation*

For the purpose of solving the wave equation a sine current is specified in Eq. (5). The magnet is assumed to be stationary. Only the vertical field component is needed and the equation is solved in dipole approximation as before. The resulting vertical field component, Γ_z, is given by

$$\Gamma_z = -\frac{b\,\mu_0}{16\,\pi^2}\, V\,\frac{\mu}{\mu_0}\, I_0\,\omega\left\{\frac{3\cos^2\theta - 1}{r^3}\cos(\omega t - kr) - \right.$$

$$\left. - k\,\frac{3\cos^2\theta - 1}{r^2}\sin(\omega t - kr) + k^2\,\frac{\sin^2\theta}{r}\cos(\omega t - kr)\right\} \tag{10}$$

$$k = \frac{\omega}{c}\ .$$

It is obvious at a glance that Eq. (10) is much more complicated than Eq. (3). The new Γ_z has 3 terms, and each of them is more complicated than the single term of Eq. (3). Moreover, a new constant, the wave number k, appears in Eq. (10). Since the speed of light, c, is very large ($c = 3 \times 10^8$ m/s), while ω for all reasonable frequencies is relatively small, k is always a very small constant. The occurrence of kr in the trigonometric functions of Eq. (10) is responsible for the wave nature of the field.

The calculations of Section 2 made the implicit assumption that gravitational fields propagate with infinite speed. In the limit when c goes to infinity and k goes to zero in Eq. (10) the result, therefore, is Eq. (3) with dI/dt replaced by $I_0 \omega \cos \omega t$ (cf. Eq. (6)).

The r–dependence of the three terms in Eq. (10) is interesting. The first term depends on $1/r^3$, the second on $1/r^2$, and the third on $1/r$. With increasing r the last two terms decrease much more slowly than the first and should, therefore, contribute more strongly to antigravity. However, since they are multiplied by the small quantities k and k^2 their net contribution at low frequencies is small. It becomes significant only at high frequencies.

Since the current varies sinusoidally, Γ_z is a function of time and so is the force after integration over the earth. With the magnet remaining stationary, the average force again turns out to be zero, confronting us with the same problem as before. This time, however, the formulas are considerably more complicated. It would require a much greater effort to discover ways and means for eliminating the time dependence if a working scheme had not been found on the basis of the first, much simpler solution. In view of its previous success it is natural to try the method of counterrotating magnets a second time. Luckily enough, the method works again, causing ωt to disappear from most of the terms in Γ_z and leading to a completely time–independent vertical component of the antigravitational force. The corresponding expression for Γ_z is given by Eq. (B41) in Appendix B.

As a result of counterrotation the traveling waves are transformed into standing waves. The wave character of the field is retained, with alternating zones of gravitational and antigravitational forces still present in the earth's interior. The zones remain stationary, their extent being governed by the magnitude of k.

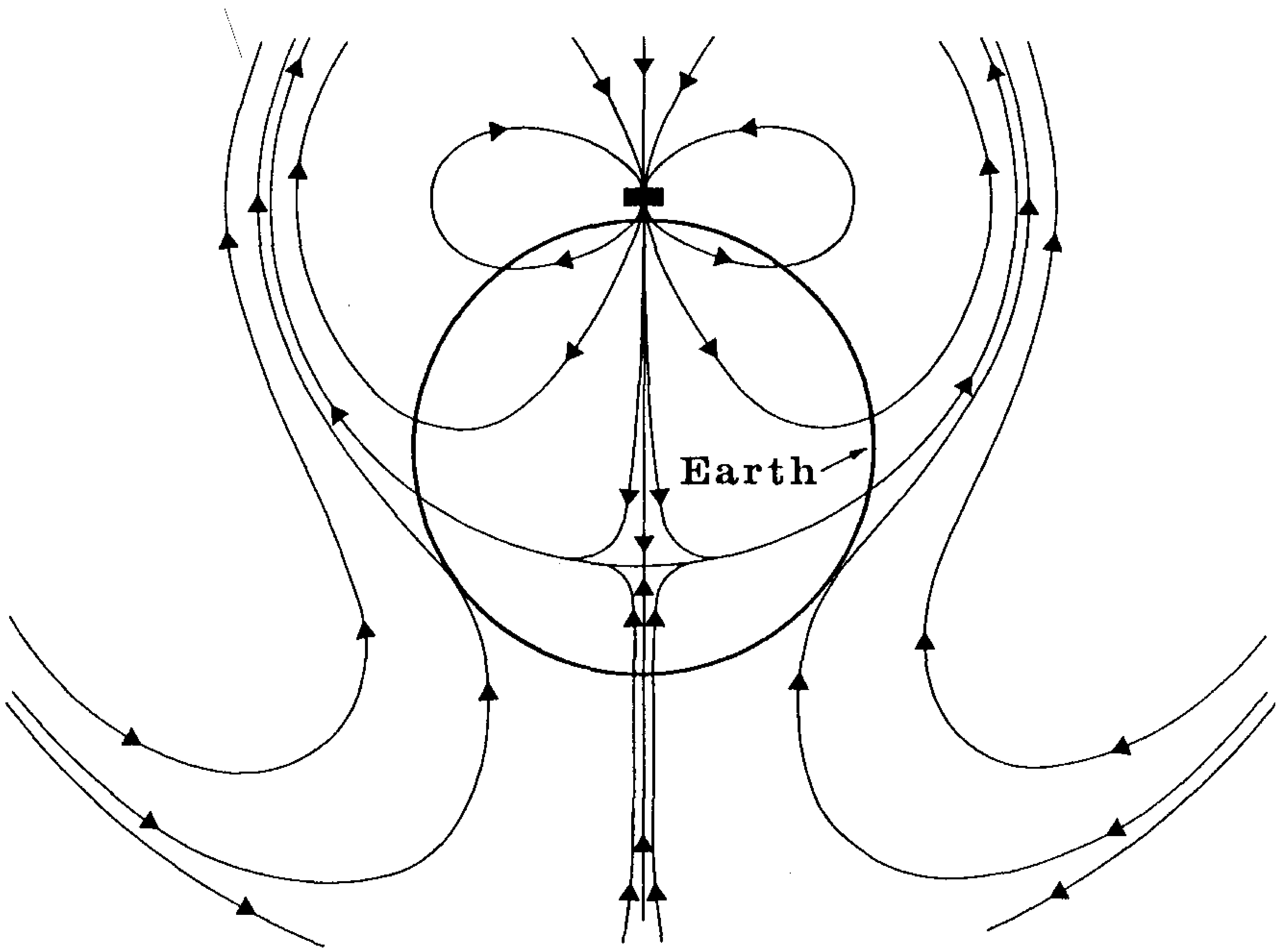

Fig. 8. Field lines in the y–z–plane produced by two magnets counterrotating at a frequency of 13 Hz. The wavelength is 2.30×10^7 m.

The field lines of Figs. 2 and 3 are now no longer correct. At distances r short compared to half a wavelength the field still retains the classical dipole shape shown in Fig. 2, but for large r the field lines change shape completely.

Figure 8 is a plot of the new field lines in a plane passing through the vertical axis and the axis of rotation (the y–z–plane). The inner zone, lying within the first circular field line, is essentially repulsive, i.e. antigravitational, as indicated by the arrows. Note that the region where the arrows point down is much narrower, and the regions where they point up are much wider, than in Fig. 2. The figure is drawn to scale for a wavelength $\lambda = 2.30 \times 10^7$ m, corresponding to the numerical example of the following section. The zone between the first circular field line and the second, partially drawn one, is essentially gravitational but lies mostly outside the earth's volume.

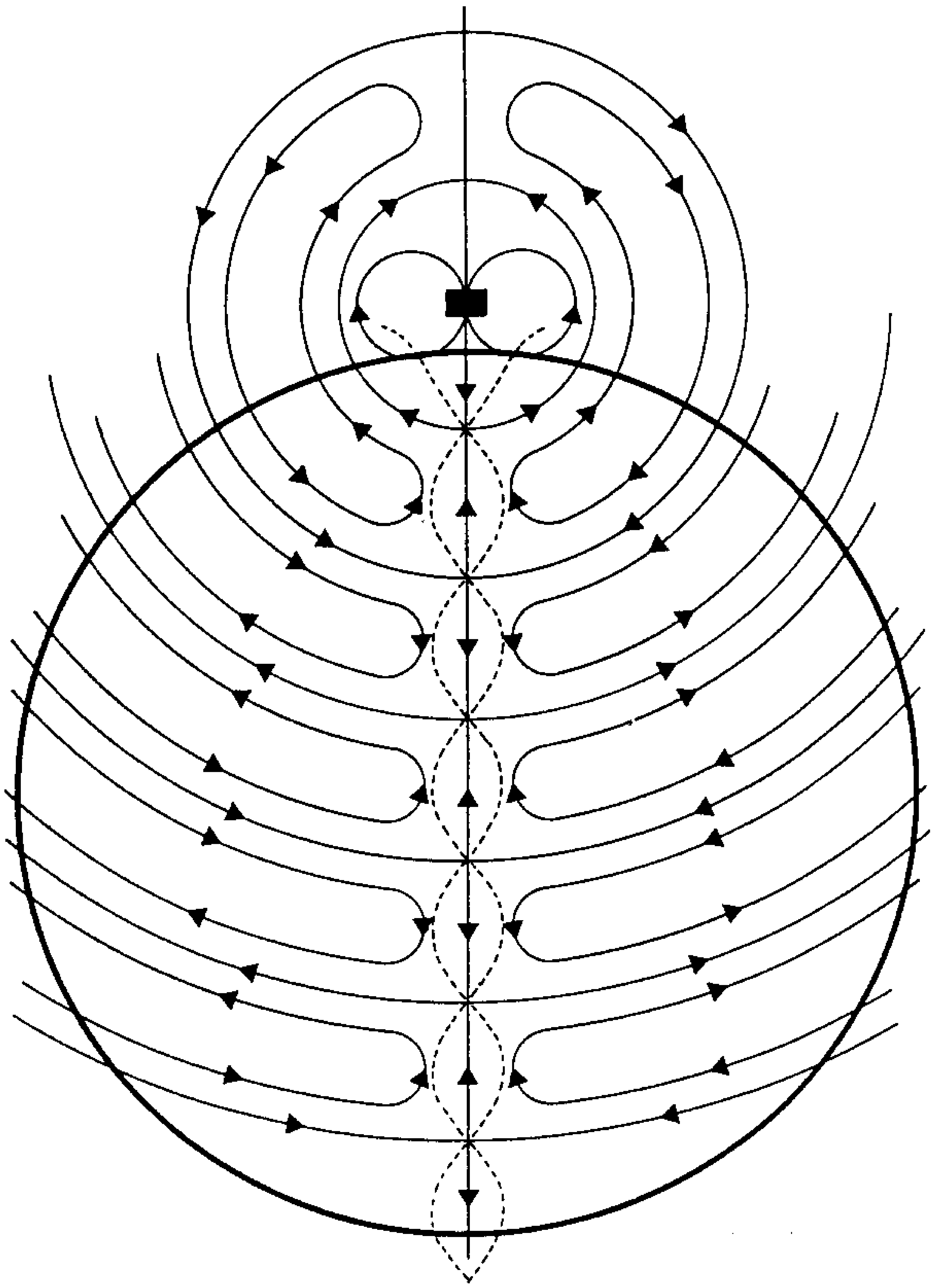

Fig. 9. Schematic view of gravitational field lines in the y–z–plane due to magnets counterrotating at 75 Hz. The wavelength is 4.0×10^6 m.

Figure 9 is a similar view of the situation when the wavelength is short. Several zones, each bounded by a circular field line, extend through the earth's interior. Gravitational and antigravitational regions alternate from one annular zone to the next, as indicated by the arrows along the vertical, although no zone is entirely repulsive or entirely attractive. The dotted curves symbolize a standing wave. Its $1/r^3$–dependence is neglected except for the steep rise near the magnet.

3.2. *The Antigravitational Force*

The final step consists, as before, in multiplying Γ_z, Eq. (B41), by the average density ρ_m and integrating the product over the earth's volume. In the result, D, the earth's diameter ($D = 1.2757 \times 10^7$ m), turns out to be a more useful parameter than the radius R. The integral contains several terms proportional to D^3, analogous to R^3 in Eq. (9), but they all cancel, leaving terms at most proportional to D^2. This results in an unfortunate loss of a factor D (about 13 million) in the formula for the antigravitational force. All terms remaining in the result are proportional either to $1/k$, $1/k^2$, or $1/k^3$. It follows that the antigravitational effect decreases with increasing k or, since k is proportional to ω, with increasing frequency or decreasing wavelength. This is precisely the effect predicted on the basis of Fig. 7.

However, the picture is not yet complete, because Eq. (B41) contains an additional ω in front of the curly brackets. After multiplying through by it the various terms in the formula for F become constant, proportional to $1/k$, and proportional to $1/k^2$, respectively. Furthermore, since $\omega/k = c$, the velocity of light, c, appears in the numerator of F. The result is a gain of $c = 3 \times 10^8$ m/s compared to a loss of $D = 1.2757 \times 10^7$ m, leading to a net gain of 23.5 s^{-1}. This may not be much, but the antigravitational force is so weak that every power of 10 counts.

Integration over the earth results in the following expression for F:

$$F = -3.555 \times 10^{-6}\, V \frac{\mu}{\mu_0} I_0\, g(k,h) \tag{11}$$

$$g(k,h) = \frac{1}{(R+h)^3}\Big\{ -D(D+2h)\big[\sin kh + \sin k(D+h)\big] + \frac{4}{k}\big[h \cos kh - (D+h)\cos k(D+h)\big] - \frac{4}{k^2}\big[\sin kh - \sin k(D+h)\big]\Big\} .$$

The minus-sign in front of the formula for F makes the force antigravitational when g(k,h) is positive.

The dependence of g(k,h) on h is so weak up to, say, $h = 100$ km that g(k,h) may be regarded as essentially independent of h up to that height. Its dependence on the wave number k for low frequencies is illustrated in Fig. 10. Both scales in the figure are enlarged by a factor 10^7. g(k,h) and k are actually of order 10^{-7}.

In Fig. 10 positive values of g(k,h) lead to antigravitation, negative ones to gravitation. When $k = 0$, i.e. at zero frequency, the effect vanishes and F is zero. Very close to $k = 0$ the force has the form of Eq. (9), which is thus seen to be valid at very low frequencies only. As k and the frequency increase, g(k,h) reaches a first antigravitational maximum at $k = 2.73 \times 10^7\ m^{-1}$ when the wavelength λ is 2.30×10^7 m or close to twice the earth's diameter. At higher frequencies g(k,h) decreases rapidly until it is zero again. As k keeps increasing, g(k,h) changes sign and becomes gravitational, reaching a negative maximum and then again going to zero at $k = 7 \times 10^7\ m^{-1}$. This oscillation continues indefinitely.

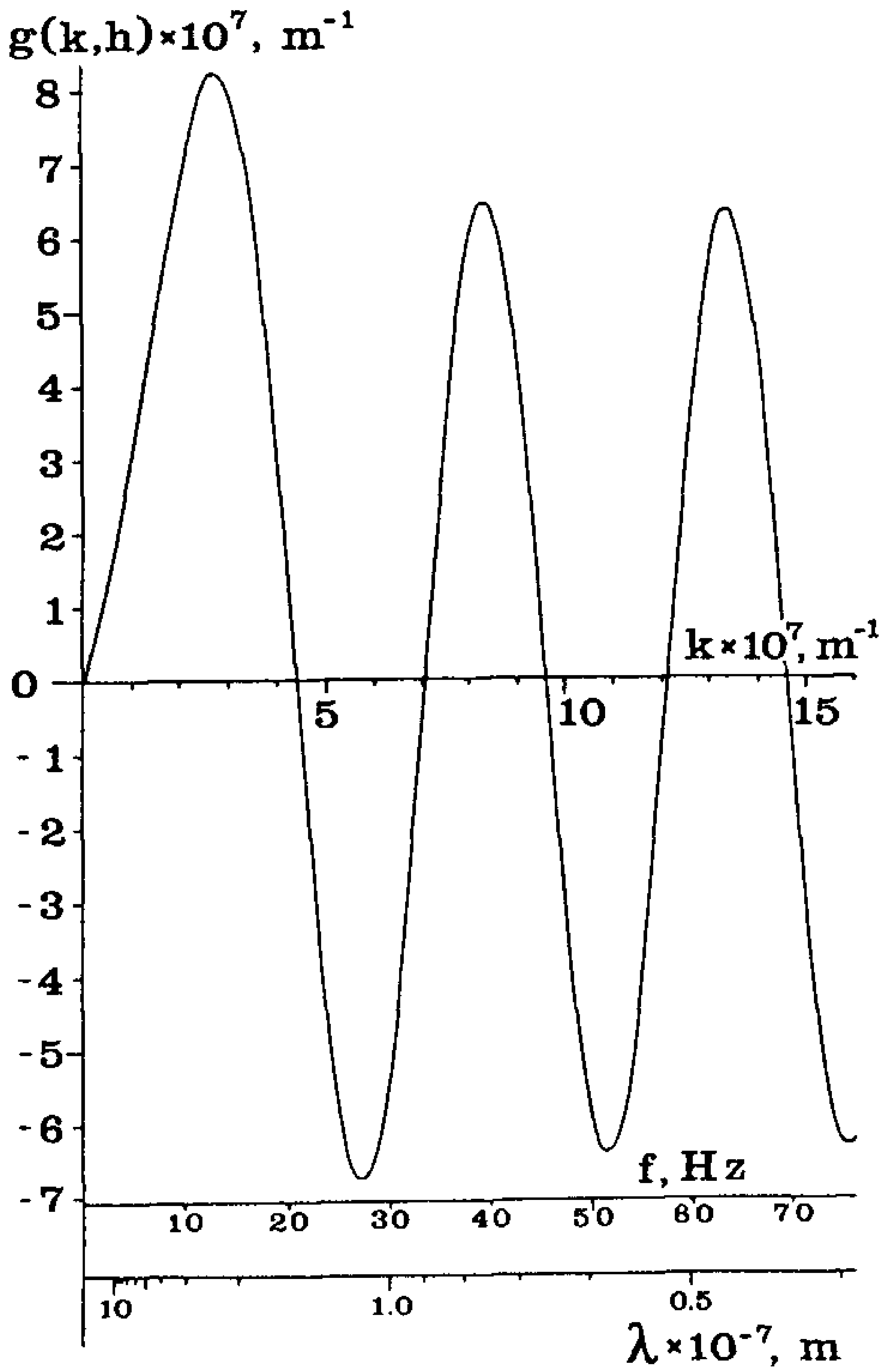

Fig. 10. The function g(k,h), Eq. (11), for $h \leq 100$ km plotted against wave number k, frequency f, and wavelength λ.

Except at extremely small values of k the expression in the first set of square brackets in Eq. (11) dominates the rest of g(k,h) because it is multiplied by the large factor $D(D + h)$. It is composed of two sine–

functions. The first of these, $\sin kh$, oscillates slowly as a function of k (or ω) while the second, $\sin k(D+h)$, oscillates rapidly. g(k,h) thus consists of a rapidly oscillating function superimposed on a slowly oscillating one, as illustrated in Fig. 11 for $h = 2'500$ km. The dotted line in the figure represents the slow oscillation. Apart from the first maximum at $k = 2.08 \times 10^{-7}\ \mathrm{m}^{-1}$, the antigravitational maxima in the curve occur at approximately $k \approx 3\pi/2h,\ 7\pi/2h,\ 11\pi/2h$, etc., or generally at $k \approx (4n+3)\pi/2h,\ n = 0, 1, 2,...$ These maxima are higher than the first maximum by a factor that is weakly dependent on h but in general is greater than 1.5. Note that the peak waves are almost entirely either in the gravitational or the antigravitational domain.

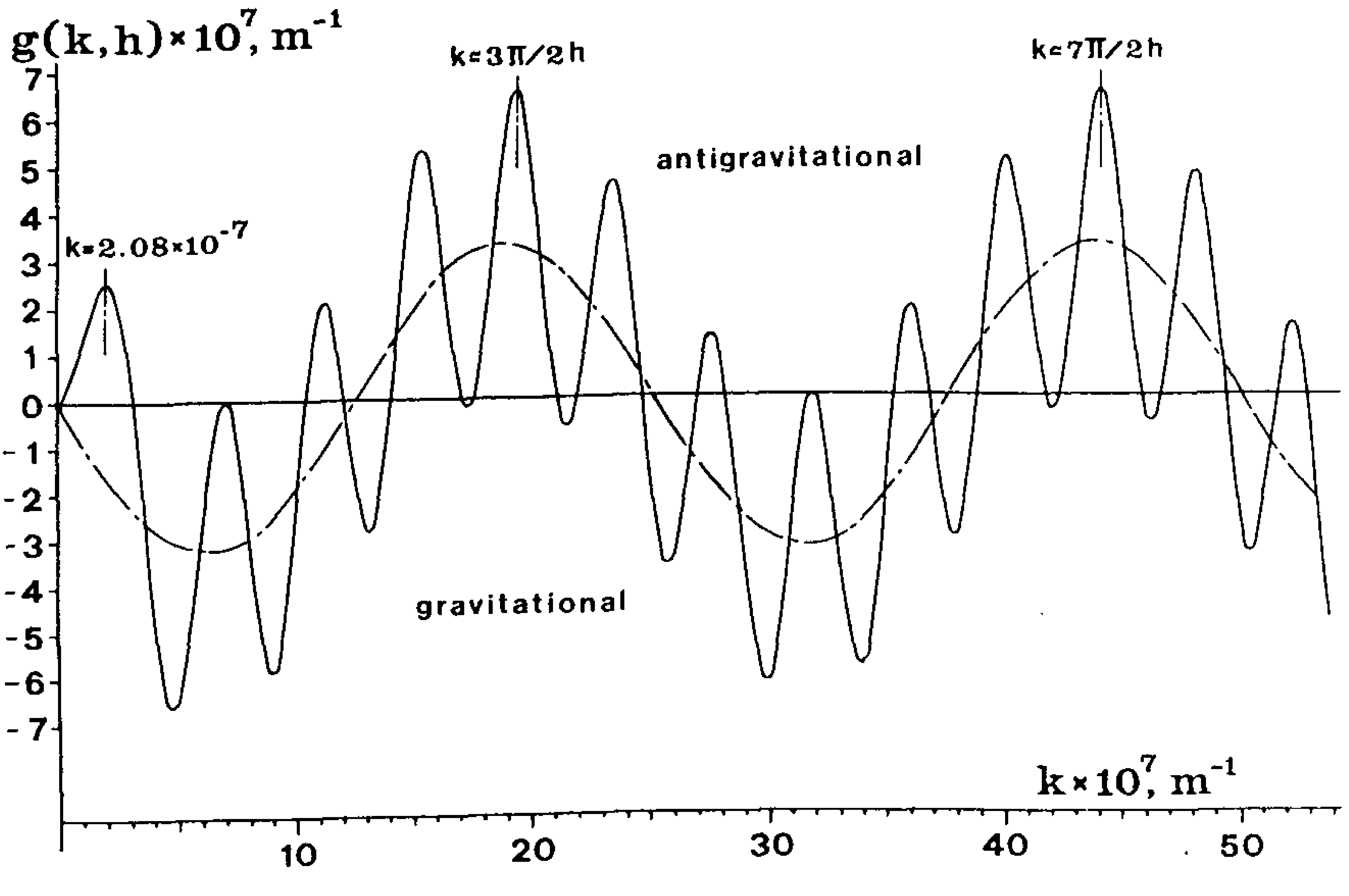

Fig. 11. The function g(k,h), Eq. (11), with $h = 2'500$ km, plotted against wave number k.

Originally, Eq. (9) led one to believe that F would rise indefinitely with growing frequency. Actually, as mentioned above, Eq. (9) is only valid for very low frequencies (or very small values of k) near zero. At higher frequencies, Eq. (11) is the correct expression for the antigravitational force. It demonstrates the impossibility of increasing F substantially by increasing the angular frequency ω. A modest increase by a factor between 1.5 and 2.5 may be achieved relative to the first peak by operating at $k \approx 3\pi/2h$ ($\omega \approx 3\pi c/2h$) or at peaks belonging to still higher frequencies, as shown in Fig. 11, but rotating magnets limit ω to fairly low values for mechanical reasons. Moreover, at its optimum value of $\omega = 3\pi c/2h$ the antigravitational force is very sensititve to small deviations of ω from the optimum. When $h = 2'500$ km, as in Fig. 11, a deviation of 5% from the optimum ω reduces the force by a factor 2. The same reduction occurs in the case of $h = 10$ km when ω varies by no more than 0.026%. Only the first peak is insensitive to variations in ω and has the further advantage of mechanical stability due to low rotational speed.

A numerical example will conclude this section. In view of the smallness of the gravitational force all factors in Eq. (11) should be made as large as possible. The values chosen for the example are,

$$\begin{aligned} V &= 1\ \mathrm{m}^3 \\ \frac{\mu}{\mu_0} &= 100'000 \\ I_0 &= 600'000 \text{ ampere turns} \ . \end{aligned} \tag{12}$$

The value of μ/μ_0 is high, but highly permeable materials are available today. I_0 is calculated on the basis of 60 amps per wire of 1 square millimeter cross–sectional area and 10'000 windings. Due to saturation effects in the permeable material the high value of μ/μ_0 is not really compatible with the large number of ampere turns, I_0. However, when dealing with the futuristic topic of antigravity one may, perhaps, be allowed to use values representing an extrapolation into the future.

Substituting the numbers of Eq. (12) into Eq. (11) gives,

$$F = -2.13 \times 10^5\, g(k,h) \ . \tag{13}$$

For g(k,h) it is best to choose the first maximum reached at a frequency of 13.03 Hz (cf. Fig. 10). This frequency of rotation is attainable even by large magnets. The maximum of the g–function at $h = 100$ m and $k = 2.73 \times 10^{-7}\ \mathrm{m}^{-1}$, corresponding to a frequency of 13.03 Hz, is (cf. Fig. 10),

lst maximum: $$g(k,h) = 8.193 \times 10^{-7}\ \mathrm{m}^{-1} \ . \tag{14}$$

When this figure is substituted into Eq. (13) the final result is,

$$F = 0.175 \text{ Newton} \tag{15}$$

or

$$F = 17.5 \text{ gram–weight} . \tag{16}$$

Thus, the antigravitational force at the first maximum is capable of lifting a total of 17.5 grams (0.617 oz). The optimum g, reached at $k = 4.7133 \times 10^{-2}\,\mathrm{m}^{-1}$ or $f = 2.25$ MHz, equals $1.254 \times 10^{-6}\,\mathrm{m}^{-1}$, leading to an optimum antigravitational force of 0.267 Newton or 26.7 grams (0.942 oz).

4. The Gravitational and Electromagnetic Fields

Underneath the rotating magnets the vertical component of the gravitational field is approximately equal to

$$\Gamma_z \approx \frac{6.73 \times 10^{-6}}{r^3} \text{ Newton/kg} . \tag{17}$$

If F in Eq. (16) could be increased to, say, 10 metric tons, which may be regarded as the approximate weight of a small aircraft, this would mean an increase of both F and Γ_z by a factor of 5.72×10^5, leading to an antigravitational field

$$\Gamma_z \approx \frac{3.85}{r^3} \text{ Newton/kg} . \tag{18}$$

The acceleration due to gravity is 10 Newton/kg, so that the field would increase the weight of any mass at a distance of $r = 1$ meter by 38.5%. An upward acceleration would need a field considerably in excess of this.

The stability of the vehicle may be a problem because of a possible tendency to turn upside down under the action of the antigravitational dipole field. A similar effect has been observed in terrestrially built flying discs such as the Avro Car, constructed by the Avro–Canada Co. in the 1950's. The instability may be eliminated by mounting, say, 3 or 4 small rotating magnets at the corners of a triangle or a square along the periphery of the craft, controlling the tilt and compensating any deviation from level flight.

As mentioned in Section 3, a strong electric field attends the induced gravitational field. With the numerical values of Eq. (12) and $f = 13.03$

Hz the electric field E underneath the craft has a magnitude approximately equal to

$$E \approx \frac{78'000}{r^3} \text{ Volt/m} . \tag{19}$$

Its direction is everywhere parallel to the gravitational field. For this reason it should be possible to focus a microwave gravitational field by focussing an electromagnetic microwave field.

If it were possible to increase F by a factor of 5.72×10^5 the electric field would increase by the same factor to

$$E \approx \frac{4.46 \times 10^{10}}{r^3} \text{ volt/m} . \tag{20}$$

Since 3 million volts/meter suffice to ionize air, such a field would lead to very strong air ionization, resulting in a brilliant glow underneath and perhaps around the craft. E decreases as $1/r^3$ for small k and r, so that the field strength falls below the ionization limit at a distance of only about 24 m.

The magnetic field outside the core, being proportional to k^2, is insignificant at low frequencies, but at the optimum frequency of 2.25 MHz its strength at a distance of 1 meter is about 1'000 Gauss. It is induced by the electro–gravitational fields and falls off as $1/r^2$. Its direction is everywhere perpendicular to the electric field.

The mesofield mentioned in the Introduction has never been observed in nature, perhaps due to its extremely weak interaction with matter. On the basis of the figures in Eq. (12) it turns out to be relatively strong, but the force it exerts on any mass is still vanishingly small, even after multiplication by 5.72×10^5. In principle, the mesofield could contribute to antigravity, but the mesofield generated by the magnets of Figs. 1 and 6 does not have the right orientation.

5. Conclusions

The theory outlined in the present study has demonstrated the feasibility of generating antigravity by means of time–dependent magnetic fields, albeit of very modest strength. Basically, this was to be expected, since otherwise the effect would have been detected long ago.

The smallness of induced antigravitation may be disappointing, but it is due in large measure to purely technological inadequacies, which some day may be overcome. New concepts could be instrumental in bringing sufficiently strong antigravitation within reach of a future technology. Electronic generation of standing antigravitational waves and the elimination of all iron cores certainly would be steps in that direction. The existence of a mathematical theory at any rate helps to eliminate guesswork and provides a basis for testing new ideas.

Some phenomena predicted by the theory correspond to effects actually observed in the vicinity of UFOs, assuming them capable of generating antigravitational fields strong enough to lift several tons. An antigravitional field is repulsive and pushes down on any object underneath the UFO. The field will bend down trees, bushes, and grasses as a UFO passes over them at low altitude. Similarly, when flying over a lake the field will cause a depression in the surface of the water. The effects have a short range due to the field's $1/r^3$–dependence and are not noticeable some distance away from the UFO. In contrast, the antigravitational force lifting the UFO has a very long range.

Of particular significance is the generation of an electric field together with the antigravitational field. If the latter is strong enough to lift a UFO, then the electric field is strong enough to ionize the surrounding air, causing a brilliant glow in the region where the field exceeds the ionization limit. Radio and television interference, dark rings around UFOs viewed through polaroid sunglasses (Roush 1968), and the prickly feeling sometimes reported by observers standing close to a UFO are further indications of the presence of a strong electric field.

When a UFO and its fields move rapidly past a stationary observer additional effects appear due to the relativistic invariance of the basic equations. The most significant of these is a magnetic pulse produced by the passing electric field. If the latter is given by Eq. (20), then the magnetic pulse has a peak strength of $v\,E/c^2$ (v = velocity of the UFO). This is not very much but still stronger than the magnetic field induced directly by the slowly rotating magnets.

As shown in Appendix B, the gravitational and electric fields generated by two counterrotating magnets have time–dependent as well as time–independent components, resulting from forming the sums and differences, $\omega t \pm \omega t$, of the rotational frequencies multiplied by the time. The time–dependent terms, therefore, oscillate with twice the frequency of rotation, or $2\omega t$ (see also Section 2.5). The most useful frequency was shown to be 13 Hz, leading to an oscillation of 26 Hz for the time–dependent field components, with a possible first harmonic at 39 Hz. Frequencies of 27 and 37 Hz have actually been observed (Schneider

1981, p.207).

The well–known vehicle interference effects caused by UFOs might be due to electric leakage fields penetrating into the space under the hood, which never is a perfect Faraday cage. Hood and chassis do not provide shielding against the gravitational field which, once inside, would induce very strong electric fields. Actually, though, the suppression of electric fields inside a Faraday cage probably goes hand in hand with a corresponding reduction of the gravitational field.

The magnetic field induced by the electro–gravitational fields at low rotational speeds is too weak to cause any measurable effects. The magnetic pulse induced relativistically by the electric field of a passing UFO might affect a compass needle but would hardly have any more significant consequences.

Clearly, there exists a considerable discrepancy between the strength of antigravitation available today and the requirement of lifting several tons. The figures of Eq. (12) are, after all, very optimistic. In reality, they may be too large by a factor of 10 or even 100. This would mean the need for increasing antigravitational forces by some 6 to 7 orders of magnitude above today's capabilities. No doubt this is a formidable task, but much room still remains for further development and improvement within the framework of the theories presented in this report or by Heim (1959).

The new effects predicted by the theory – antigravity is only one of them – are near the limit of measurability, so that experimental verification is difficult to accomplish. Some experiments have been suggested (Heim 1985, Auerbach 1983, 1985, Harasim et al. 1985), but so far none of them have been carried out. Perhaps the prospect of achieving practical results might serve as an inducement for performing an experiment in the not too distant future.

Acknowledgments

The author wishes to express his thanks to I. von Ludwiger for encouraging this research and to H.–M. Friess for giving valuable help and advice on the computer–aided editing of the manuscript. He gratefully acknowledges several interesting and helpful discussions with I. von Ludwiger, W. Bucher, H.–M. Friess, A. Müller, and A. Schneider, all of MUFON–CES. He also wishes to thank A. Harasim and A. Müller for reading the manuscript and suggesting improvements.

Appendix A

In this appendix a set of equations is derived describing a 4–dimensional unified field theory of gravitation and electromagnetism. The derivation is based entirely on Heim's concepts as presented in the first volume of "Elementarstrukturen der Materie" (Heim 1989) and in a number of articles by I. von Ludwiger (1976, 1981). In his writings, Heim never presents either an explicit derivation of the equations or the equations themselves. Hence the need for deriving them in this appendix. Various remarks in Heim's book are used as guides in the process. Actually, though, Heim's theory is 6–dimensional, and a 4–dimensional version is at best an approximation. The results are presented nevertheless, since nothing else is available at the moment.

The basis of the derivation is an analogy between the gravitational field equations and Maxwell's equations. The two systems have different metrics, so that one coordinate transformation leaves Maxwell's equations invariant, while another leaves the gravitational equations invariant. Both systems of equations are represented by tensor divergences, and the idea is to find a linear combination of gravitational and electromagnetic tensor components which remain invariant under a combined coordinate transformation in Minkowski space.

1. *Maxwell's Equations*

Maxwell's equations in vacuum may be written in the following symmetric form:

$$\nabla \cdot \sqrt{\varepsilon_0}\,\mathbf{E} = c\sqrt{\mu_0}\,\rho_e \tag{A1a}$$

$$\nabla \times \sqrt{\varepsilon_0}\,\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\sqrt{\mu_0}\,\mathbf{H} \tag{A1b}$$

$$\nabla \cdot \sqrt{\mu_0}\,\mathbf{H} = 0 \tag{A1c}$$

$$\nabla \times \sqrt{\mu_0}\,\mathbf{H} = \sqrt{\mu_0}\,\mathbf{j}_e + \frac{1}{c}\frac{\partial}{\partial t}\sqrt{\varepsilon_0}\,\mathbf{E} \tag{A1d}$$

$$\sqrt{\varepsilon_0 \mu_0} = \frac{1}{c}\ . \tag{A1e}$$

In addition, one has to satisfy the equation of continuity,

$$\nabla \cdot \mathbf{j}_e + \frac{\partial \rho_e}{\partial t} = 0\ , \tag{A2}$$

and the Lorentz force density in vacuum,

$$\mathbf{f}_e = \rho_e(\mathbf{E} + \mathbf{v}\times\mu_0\mathbf{H})\ . \tag{A3}$$

The index e in these equations stands for "electrical".

The system of equations (A1)–(A3) may be represented by the divergences of an antisymmetric tensor $T_e^{\mu\nu}$, a source s_e^{μ}, the dual tensor $^*T_e^{\mu\nu}$, and the dual source q_e^{μ}:

$$T_e^{\mu\nu} = \begin{pmatrix} 0 & \sqrt{\mu_0}\,H_z & -\sqrt{\mu_0}\,H_y & -i\sqrt{\varepsilon_0}\,E_x \\ -\sqrt{\mu_0}\,H_z & 0 & \sqrt{\mu_0}\,H_x & -i\sqrt{\varepsilon_0}\,E_y \\ \sqrt{\mu_0}\,H_y & -\sqrt{\mu_0}\,H_x & 0 & -i\sqrt{\varepsilon_0}\,E_z \\ i\sqrt{\varepsilon_0}\,E_x & i\sqrt{\varepsilon_0}\,E_y & i\sqrt{\varepsilon_0}\,E_z & 0 \end{pmatrix} \tag{A4}$$

$$s_e^{\mu} = [\sqrt{\mu_0}\,j_{ex}, \sqrt{\mu_0}\,j_{ey}, \sqrt{\mu_0}\,j_{ez}, ic\sqrt{\mu_0}\,\rho_e] \tag{A5}$$

$$^*T_e^{\mu\nu} = \begin{pmatrix} 0 & -i\sqrt{\varepsilon_0}\,E_z & i\sqrt{\varepsilon_0}\,E_y & \sqrt{\mu_0}\,H_x \\ i\sqrt{\varepsilon_0}\,E_z & 0 & -i\sqrt{\varepsilon_0}\,E_x & \sqrt{\mu_0}\,H_y \\ -i\sqrt{\varepsilon_0}\,E_y & i\sqrt{\varepsilon_0}\,E_x & 0 & \sqrt{\mu_0}\,H_z \\ -\sqrt{\mu_0}\,H_x & -\sqrt{\mu_0}\,H_y & -\sqrt{\mu_0}\,H_z & 0 \end{pmatrix} \tag{A6}$$

$$q_e^{\mu} = [0, 0, 0, 0]\ . \tag{A7}$$

The 6 equations (A1a)–(A3) are obtained from the following operations on the tensors and sources of Eqs. (A4)–(A7):

$$\frac{\partial}{\partial x_e^{\nu}} T_e^{\mu\nu} = s_e^{\mu} \qquad \text{leads to Eqs. (A1a) and (A1d)} \tag{A8a}$$

$$\frac{\partial}{\partial x_e^{\nu}} {}^*T_e^{\mu\nu} = q_e^{\mu} \qquad \text{leads to Eqs. (A1b) and (A1c)} \tag{A8b}$$

$$\frac{\partial}{\partial x_e^{\mu}} s_e^{\mu} = 0 \qquad \text{leads to Eq. (A2)} \tag{A8c}$$

$$f_e^{j} = T_e^{j\nu} s_e^{\nu} \qquad \text{leads to Eq. (A3)}\ . \tag{A8d}$$

Greek indices run from 1 to 4, Latin ones from 1 to 3. The summation convention is used throughout, except where stated. The 4 values of x_e^{μ} are,

$$x_e^1 = x, \quad x_e^2 = y, \quad x_e^3 = z, \quad x_e^4 = ict \; . \tag{A9}$$

Maxwell's equations are valid in Minkowski space, which Heim denotes by R_{-4} (Heim 1989). Its coordinates are given in Eq. (A9). The equations above are invariant under a Lorentz transformation, i.e. if the transformed coordinates are given by

$$\bar{x}_e^{\mu} = A_{e\mu\nu} x_e^{\nu} \tag{A10}$$

and vice versa,

$$x_e^{\mu} = A_{e\mu\nu}^{-1} \bar{x}_e^{\nu} \; , \tag{A11}$$

then $A_{e\mu\nu}$ and $A_{e\mu\nu}^{-1}$ are elements of the following 4×4 matrices:

$$A_e = \begin{pmatrix} \gamma_- & 0 & 0 & i\frac{v}{c}\gamma_- \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\frac{v}{c}\gamma_- & 0 & 0 & \gamma_- \end{pmatrix}, \quad A_e^{-1} = \begin{pmatrix} \gamma_- & 0 & 0 & -i\frac{v}{c}\gamma_- \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\frac{v}{c}\gamma_- & 0 & 0 & \gamma_- \end{pmatrix} \tag{A12}$$

with

$$\gamma_- = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{A13}$$

for a translation in the x–direction. The matrix products are $A_e^{-1} A_e = A_e A_e^{-1} = \mathbf{I}$ (unit matrix), or

$$A_{e\mu\lambda} A_{e\lambda\nu}^{-1} = A_{e\mu\lambda}^{-1} A_{e\lambda\nu} = \delta_{\mu\nu} \; . \tag{A14}$$

Heim denotes the transformation matrix A_e by $\hat{A}_-$.

2. *The Gravitational Equations*

The gravitational equations are similar in form to Maxwell's equations. The gravitational vector $\alpha\mathbf{\Gamma}$, where $\mathbf{\Gamma}$ is the gravitational field, corresponds to the electric displacement vector in vacuum, $\mathbf{D} = \varepsilon_0\mathbf{E}$. The magnetic induction in vacuum, $\mathbf{B} = \mu_0\mathbf{H}$, is replaced by the vector $\beta\boldsymbol{\mu}$, where $\boldsymbol{\mu}$ is the gravitational mesofield, whose relation to $\mathbf{\Gamma}$ is similar to the relation of $\mathbf{H}$ to $\mathbf{E}$. According to Heim, the gravitational equations are valid in R_{+4}, which differs from R_{-4} in that $x_4 = \omega t$ and not ict. ω is an as yet undetermined velocity, which obeys the relation,

$$\omega = \frac{1}{\sqrt{\alpha\beta}} . \qquad \text{(A15)}$$

This results in a change of sign on the right–hand side of the gravitational equation whose equivalent is Eq. (A1b). Finally, $\beta\boldsymbol{\mu}$ differs from $\mathbf{B}$ in having as source the field mass density $\rho_m - \rho_{m0}$, while $\mathbf{B}$ is source–free due to the non–existence of magnetic monopoles.

With these changes and analogies the gravitational equations in Heim's symmetric notation become (v. Ludwiger 1975),

$$\nabla\cdot\sqrt{\alpha}\,\mathbf{\Gamma} = \omega\sqrt{\beta}\,\rho_m \qquad \text{(A16a)}$$

$$\nabla\times\sqrt{\alpha}\,\mathbf{\Gamma} = \frac{1}{\omega}\frac{\partial}{\partial t}\sqrt{\beta}\,\boldsymbol{\mu} \qquad \text{(A16b)}$$

$$\nabla\cdot\sqrt{\beta}\,\boldsymbol{\mu} = \sqrt{\beta}\,\omega\,(\rho_m - \rho_{m0}) \qquad \text{(A16c)}$$

$$\nabla\times\sqrt{\beta}\,\boldsymbol{\mu} = \sqrt{\beta}\,\mathbf{j}_m + \frac{1}{\omega}\frac{\partial}{\partial t}\sqrt{\alpha}\,\mathbf{\Gamma} . \qquad \text{(A16d)}$$

Here the subscript m stands for "mass", ρ_m is the total mass density, ρ_{m0} is the particle mass density, and $\mathbf{j}_m$ is the mass current density. Added to this is the continuity equation

$$\nabla\cdot\mathbf{j}_m + \frac{\partial\rho_m}{\partial t} = 0 . \qquad \text{(A17)}$$

The force density will be derived from the tensor relations.

The tensors and sources whose divergences lead to Eqs. (A16a)–(A17) are

$$T_g^{\mu\nu} = \begin{pmatrix} 0 & \sqrt{\beta}\,\mu_z & -\sqrt{\beta}\,\mu_y & -\sqrt{\alpha}\,\Gamma_x \\ -\sqrt{\beta}\,\mu_z & 0 & \sqrt{\beta}\,\mu_x & -\sqrt{\alpha}\,\Gamma_y \\ \sqrt{\beta}\,\mu_y & -\sqrt{\beta}\,\mu_x & 0 & -\sqrt{\alpha}\,\Gamma_z \\ \sqrt{\alpha}\,\Gamma_x & \sqrt{\alpha}\,\Gamma_y & \sqrt{\alpha}\,\Gamma_z & 0 \end{pmatrix} \tag{A18}$$

$$s_g^{\mu} = [\sqrt{\beta}\,j_{mx}, \sqrt{\beta}\,j_{my}, \sqrt{\beta}\,j_{mz}, \ \omega\sqrt{\beta}\,\rho_m] \ . \tag{A19}$$

$$^*T_g^{\mu\nu} = \begin{pmatrix} 0 & -\sqrt{\alpha}\,\Gamma_z & \sqrt{\alpha}\,\Gamma_y & \sqrt{\beta}\,\mu_x \\ \sqrt{\alpha}\,\Gamma_z & 0 & -\sqrt{\alpha}\,\Gamma_x & \sqrt{\beta}\,\mu_y \\ -\sqrt{\alpha}\,\Gamma_y & \sqrt{\alpha}\,\Gamma_x & 0 & \sqrt{\beta}\,\mu_z \\ -\sqrt{\beta}\,\mu_x & -\sqrt{\beta}\,\mu_y & -\sqrt{\beta}\,\mu_z & 0 \end{pmatrix} \tag{A20}$$

$$q_g^{\mu} = [0, \ 0, \ 0, \ -\omega\sqrt{\beta}(\rho_m - \rho_{m0})] \ . \tag{A21}$$

The subscript g stands for "gravitational".

The relations leading to the gravitational equations above are

$$\frac{\partial}{\partial x_g^{\nu}} T_g^{\mu\nu} = s_g^{\mu} \quad \text{leads to Eqs. (A16a) and (A16d)} \tag{A22a}$$

$$\frac{\partial}{\partial x_g^{\nu}} {}^*T_g^{\mu\nu} = q_g^{\mu} \quad \text{leads to Eqs. (A16b) and (A16c)} \tag{A22b}$$

$$\frac{\partial}{\partial x_g^{\mu}} s_g^{\mu} = 0 \quad \text{leads to Eq. (A17)} \ . \tag{A22c}$$

In analogy to Eq. (A8d) the force density is derived from the relation

$$f_g^{j} = T_g^{j\nu} s_g^{\nu} \ , \tag{A22d}$$

resulting in the expression

$$\mathbf{f}_g = \rho_m (- \mathbf{\Gamma} + \mathbf{v} \times \beta \boldsymbol{\mu}) \ . \tag{A23}$$

In the equations above,

$$x_g^1 = x, \quad x_g^2 = y, \quad x_g^3 = z, \quad x_g^4 = \omega t \; . \tag{A24}$$

The coordinates transform as

$$\overline{x}_g^{\mu} = A_{g\mu\nu} x_g^{\nu} \tag{A25a}$$

$$x_g^{\mu} = A_{g\mu\nu}^{-1} \overline{x}_g^{\nu} \; , \tag{A25b}$$

with the transformation matrices given by

$$A_g = \begin{pmatrix} \gamma_+ & 0 & 0 & \frac{v}{\omega}\gamma_+ \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{v}{\omega}\gamma_+ & 0 & 0 & \gamma_+ \end{pmatrix}, \quad A_g^{-1} = \begin{pmatrix} \gamma_+ & 0 & 0 & -\frac{v}{\omega}\gamma_+ \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{v}{\omega}\gamma_+ & 0 & 0 & \gamma_+ \end{pmatrix} \tag{A26}$$

and

$$\gamma_+ = \frac{1}{\sqrt{1 + \frac{v^2}{\omega^2}}} \tag{A27a}$$

$$A_{g\mu\lambda} A_{g\lambda\nu}^{-1} = A_{g\mu\lambda}^{-1} A_{g\lambda\nu} = \delta_{\mu\nu} \; . \tag{A27b}$$

Heim denotes the A_g–matrix by $\hat{A}_+$.

3. *The Unified Field Tensor* $T^{\mu\nu}$

According to Heim, neither R_{-4} nor R_{+4} are true representations of reality. Instead, R_{-4} and R_{+4} are to be combined into a common space R_4 in which (a) $x^4 = ict$, and (b) the field equations are invariant with respect to the combined transformation $B = A_e A_g$. The corresponding tensor, $T^{\mu\nu}$, is a linear combination of $T_e^{\mu\nu}$ and $T_g^{\mu\nu}$ with coefficients to be determined (v. Ludwiger 1981, p. 113). Since $x^4 = ict$ the new equations are again valid in Minkowski space, as are Maxwell's equations, so that $T_e^{\mu\nu}$ need not be changed, i.e. its coefficient in the combined tensor may be set equal to unity. The ansatz for $T^{\mu\nu}$ and s^{μ} then becomes,

$$T^{\mu\nu} = T_e^{\mu\nu} + b_{(\mu\nu)} T_g^{\mu\nu} \tag{A28a}$$

$$s^{\mu} = s_e^{\mu} + p_{(\mu)} s_g^{\mu} \tag{A28b}$$

$$x^{\mu} = x_e^{\mu} \quad \text{(cf. Eq. (A9))} \ . \tag{A28c}$$

Parantheses around some of the lower indices indicate that the summation convention is suspended.

The new equations should again be of the form

$$\frac{\partial T^{\mu\nu}}{\partial x_e^{\nu}} = s^{\mu} \tag{A29a}$$

$$\frac{\partial^* T^{\mu\nu}}{\partial x_e^{\nu}} = q_g^{\mu} \tag{A29b}$$

$$\frac{\partial s^{\mu}}{\partial x_e^{\mu}} = 0 \tag{A29c}$$

$$f^j = T^{j\nu} s^{\nu} \ . \tag{A29d}$$

The new transformation matrices $B = A_e A_g$ and $B^{-1} = A_g^{-1} A_e^{-1}$ are

$$B_{\mu\nu} = A_{e\mu\lambda} A_{g\lambda\nu} \tag{A30a}$$

$$B^{-1}_{\mu\nu} = A^{-1}_{g\mu\kappa} A^{-1}_{e\kappa\nu} \ . \tag{A30b}$$

Equation (A29a) can be shown to be invariant under the combined transformation B.

Next, Eqs. (A28a) and (A28b) are substituted into Eq. (A29a), giving

$$\frac{\partial}{\partial x_e^{\nu}} \left(T_e^{\mu\nu} + b_{(\mu\nu)} T_g^{\mu\nu} \right) = s_e^{\mu} + p_{(\mu)} s_g^{\mu} \ . \tag{A31}$$

$T_e^{\mu\nu}$ is a solution of Eq. (A8a), eliminating the first terms on the left– and right–hand sides of Eq. (A31) and leaving the relation

$$b_{(\mu\nu)} \frac{\partial}{\partial x_e^{\nu}} T_g^{\mu\nu} = p_{(\mu)} s_g^{\mu} \ . \tag{A32}$$

The coordinates x_e^j, $j = 1,2,3$, are the same as x_g^j, so that according to Eq. (A22a),

$$b_{ij} = p_j = 1 \ . \tag{A33}$$

Only the terms in $\mu = 4$ and $\nu = 4$ require adjustment. Writing out in full the 4 equations resulting from Eq. (A32) for $\mu = 1$, 2, 3, and 4, and using Eqs. (A18) and (A19), gives

$$\mu = 1: \qquad \sqrt{\beta}\left(\frac{\partial \mu_z}{\partial y} - \frac{\partial \mu_y}{\partial z}\right) - b_{14}\frac{\sqrt{\alpha}}{ic}\frac{\partial \Gamma_x}{\partial t} = \sqrt{\beta}\, j_{mx} \tag{A34a}$$

$$\mu = 2: \qquad \sqrt{\beta}\left(\frac{\partial \mu_x}{\partial z} - \frac{\partial \mu_z}{\partial x}\right) - b_{24}\frac{\sqrt{\alpha}}{ic}\frac{\partial \Gamma_y}{\partial t} = \sqrt{\beta}\, j_{my} \tag{A34b}$$

$$\mu = 3: \qquad \sqrt{\beta}\left(\frac{\partial \mu_y}{\partial x} - \frac{\partial \mu_x}{\partial y}\right) - b_{34}\frac{\sqrt{\alpha}}{ic}\frac{\partial \Gamma_z}{\partial t} = \sqrt{\beta}\, j_{mz} \tag{A34c}$$

$$\mu = 4: \qquad \sqrt{\alpha}\left(b_{41}\frac{\partial \Gamma_x}{\partial x} + b_{42}\frac{\partial \Gamma_y}{\partial y} + b_{43}\frac{\partial \Gamma_z}{\partial z}\right) = p_4\,\omega\sqrt{\beta}\,\rho_m \ . \tag{A34d}$$

Comparing Eqs. (A34a) – (A34c) with Eq. (A16d) shows that

$$b_{14} = b_{24} = b_{34} = \frac{ic}{\omega} \ . \tag{A35}$$

$T^{\mu\nu}$ must be an antisymmetric tensor, implying the relation

$$b_{41} = b_{42} = b_{43} = \frac{ic}{\omega} \ . \tag{A36}$$

Using these values of $b_{\mu\nu}$, Eq. (A34d) becomes

$$\frac{ic}{\omega}\nabla\cdot\sqrt{\alpha}\,\Gamma = p_4\,\omega\sqrt{\beta}\,\rho_m \ . \tag{A37}$$

When this is compared with Eq. (A16a) the last undetermined coefficient, p_4, is seen to be

$$p_4 = \frac{ic}{\omega} \ . \tag{A38}$$

With all coefficients known, the results may now be substituted into Eqs. (A28a) and (A28b) from which the unified field tensors are derived. The result is best expressed in terms of the following quantities introduced by Heim:

$$\mathbf{G} = \sqrt{\varepsilon_o}\,\mathbf{E} + \frac{c}{\omega}\sqrt{\alpha}\,\boldsymbol{\Gamma} \tag{A39a}$$

$$\mathbf{C} = \sqrt{\mu_o}\,\mathbf{H} + \sqrt{\beta}\,\boldsymbol{\mu} \tag{A39b}$$

$$\mathbf{j} = \sqrt{\mu_o}\,\mathbf{j}_e + \sqrt{\beta}\,\mathbf{j}_m \tag{A39c}$$

$$\rho = \sqrt{\mu_o}\,\rho_e + \sqrt{\beta}\,\rho_m \ . \tag{A39d}$$

In terms of the quantities above the unified field tensor and its source are,

$$T^{\mu\nu} = \begin{pmatrix} 0 & C_z & -C_y & -iG_x \\ -C_z & 0 & C_x & -iG_y \\ C_y & -C_x & 0 & -iG_z \\ iG_x & iG_y & iG_z & 0 \end{pmatrix} \tag{A40}$$

$$s^{\mu} = \left[j_x,\ j_y,\ jy_z,\ ic\rho \right] \ . \tag{A41}$$

The dual tensor and its source are

$$^{*}T^{\mu\nu} = \begin{pmatrix} 0 & -iG_z & iG_y & C_x \\ iG_z & 0 & -iG_x & C_y \\ -iG_y & iG_x & 0 & C_z \\ -C_x & -C_y & -C_z & 0 \end{pmatrix} \tag{A42}$$

$$q^{\mu} = [\,0,\ 0,\ 0,\ -\omega\sqrt{\beta}(\rho_m - \rho_{m0})] \ . \tag{A43}$$

The unified field equations are obtained by substituting Eqs. (A40) – (A43) into Eqs. (A29a) and (A29b). The result is

$$\nabla \cdot \mathbf{G} = c\,\rho \tag{A44a}$$

$$\nabla \times \mathbf{G} = -\frac{1}{c}\frac{\partial \mathbf{C}}{\partial t} \tag{A44b}$$

$$\nabla \cdot \mathbf{C} = \omega\sqrt{\beta}(\rho_m - \rho_{m0}) \tag{A44c}$$

$$\nabla \times \mathbf{C} = \mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{G}}{\partial t} \ . \tag{A44d}$$

The equation of continuity, derived from Eq. (A29c), is

$$\nabla\cdot\mathbf{j} + \frac{\partial\rho}{\partial t} = 0 \ . \tag{A45}$$

Finally, the force density is obtained from Eq. (A29d),

$$\mathbf{f} = c\rho\mathbf{G} + \mathbf{j}\times\mathbf{C} \ . \tag{A46}$$

Two of the 3 constants, α, β, and ω still remain to be determined. Consider first Eq. (A44a). Written out in full it becomes

$$\nabla\cdot\sqrt{\varepsilon_0}\,\mathbf{E} + \frac{c}{\omega}\sqrt{\alpha}\,\nabla\cdot\boldsymbol{\Gamma} = c\sqrt{\mu_0}\,\rho_e + c\sqrt{\beta}\,\rho_m \ . \tag{A47}$$

In the absence of both electric charges and electric fields the equation reduces to

$$\frac{1}{\omega}\sqrt{\alpha}\,\nabla\cdot\boldsymbol{\Gamma} = \sqrt{\beta}\,\rho_m \tag{A48}$$

or

$$\nabla\cdot\boldsymbol{\Gamma} = \frac{1}{\alpha}\,\rho_m \tag{A49}$$

by use of Eq. (A15). Since $\boldsymbol{\Gamma}$ is the gravitational field, Eq. (A49) must correspond to the well–known expression for the divergence of that field,

$$\nabla\cdot\boldsymbol{\Gamma} = 4\pi\gamma\rho_m \ , \tag{A50}$$

from which it is evident that

$$\alpha = \frac{1}{4\pi\gamma} \ . \tag{A51}$$

Next, consider Eq. (A46). Again, writing it out in full results in the expression,

$$\mathbf{f} = \rho_e\mathbf{E} + \frac{c^2}{\omega^2}\rho_m\boldsymbol{\Gamma} + \mathbf{j}_e\times\mathbf{B} + \beta\,\mathbf{j}_m\times\boldsymbol{\mu} + c\sqrt{\beta\varepsilon_0}\,\rho_m\mathbf{E} +$$

$$+ \frac{c^2}{\omega}\sqrt{\alpha\mu_0}\,\rho_e\boldsymbol{\Gamma} + \sqrt{\beta\mu_0}\,(\mathbf{j}_m\times\mathbf{H} + \mathbf{j}_e\times\boldsymbol{\mu}) \ . \tag{A52}$$

The second term on the right must be the usual force density $\rho_m\boldsymbol{\Gamma}$ due to the gravitational field. This implies that $c/\omega = 1$ or

$$\omega = c \ . \tag{A53}$$

The meaning of the 5th and 6th terms in Eq. (A52) is not clear. The coefficient $c\sqrt{\beta\varepsilon_o} \approx 10^{-10}$ is small, so that this term may have escaped detection, but $c\sqrt{\alpha\mu_o}$ (with $\omega = c$) is of order 10^{10}. Since such a strong coupling of the charge density to the gravitational field is completely at variance with observation, it must be assumed that this particular cross term is zero. This may be explained by assuming that Γ never directly acts on charges. The last two terms in Eq. (A52) represent interactions between mass current density and magnetic field on the one hand, and between electric current density and the gravitational mesofield on the other. The coefficient $\sqrt{\beta\mu_0}$ is of order 10^{-16} and hence extremely small, so that the contribution of these terms to the total force would usually be of negligible magnitude. Only a very sen–sitive experiment could detect the terms, provided they exist at all.

Finally, with use of Eq. (A15) β is given by

$$\beta = \frac{1}{\alpha c^2} \ . \tag{A54}$$

Using Eq. (A53) the 4 quantities of Eq. (A39) become

$$\mathbf{G} = \sqrt{\varepsilon_0}\,\mathbf{E} + \sqrt{\alpha}\,\mathbf{\Gamma} \tag{A55a}$$

$$\mathbf{C} = \sqrt{\mu_0}\,\mathbf{H} + \sqrt{\beta}\,\boldsymbol{\mu} \tag{A55b}$$

$$\mathbf{j} = \sqrt{\mu_0}\,\mathbf{j}_e + \sqrt{\beta}\,\mathbf{j}_m \tag{A55c}$$

$$\rho = \sqrt{\mu_0}\,\rho_e + \sqrt{\beta}\,\rho_m \ . \tag{A55d}$$

The full set of unified field equations, written out in the usual notation and including the 3 small cross terms of Eq. (A52), is

$$\nabla\cdot\mathbf{E} = \frac{1}{\varepsilon_0}\rho_e \tag{A56a}$$

$$\nabla\cdot\mathbf{\Gamma} = \frac{1}{\alpha}\rho_m \tag{A56b}$$

$$\nabla\times\left(\mathbf{E} + \sqrt{\frac{\alpha}{\varepsilon_0}}\,\mathbf{\Gamma}\right) = -\mu_0\frac{\partial}{\partial t}\left(\mathbf{H} + \sqrt{\frac{\beta}{\mu_0}}\,\boldsymbol{\mu}\right) \tag{A56c}$$

$$\nabla\cdot\mathbf{H} = 0 \tag{A56d}$$

$$\nabla\cdot\boldsymbol{\mu} = c(\rho_m - \rho_{m0}) \tag{A56e}$$

$$\nabla \times \left(\mathbf{H} + \sqrt{\frac{\beta}{\mu_0}}\, \boldsymbol{\mu} \right) = \mathbf{j}_e + \sqrt{\frac{\beta}{\mu_0}}\, \mathbf{j}_m + \varepsilon_0 \frac{\partial}{\partial t} \left(\mathbf{E} + \sqrt{\frac{\alpha}{\varepsilon_0}}\, \boldsymbol{\Gamma} \right) \qquad \text{(A56f)}$$

$$\mathbf{f} = \rho_e \mathbf{E} + \rho_m \boldsymbol{\Gamma} + \mathbf{j}_e \times \mathbf{B} + \beta\, \mathbf{j}_m \times \boldsymbol{\mu} + c\sqrt{\beta \varepsilon_0}\, \rho_m \mathbf{E} + \\ + \sqrt{\beta \mu_0}\, (\mathbf{j}_m \times \mathbf{H} + \mathbf{j}_e \times \boldsymbol{\mu}) \; . \qquad \text{(A56g)}$$

The fact that Eq. (A56c) does not reduce to Eq. (A16b) in the absence of electromagnetic fields is a consequence of the fact that Eq. (A56) exists in Minkowski space, R_{-4}, with $x^4 = ict$, whereas Eq. (A16b) exists in R_{+4} with $x^4 = ct$ $(\omega = c)$.

Appendix B

1. *The Field Equations*

In the 4–dimensional unified field theory derived in Appendix A anti–gravitational phenomena are described by Eqs. (A56c) and (A56f). In the first of these, $\boldsymbol{\mu}$ may be neglected relative to the much stronger impressed magnetic field $\mu_o \mathbf{H} = \mathbf{B}_o$. In the second equation $\mathbf{j}_m$ is zero because no mass currents are present. Finally, $\mathbf{j}_e$ may be disregarded because it is already incorporated into $\mathbf{B}_o$. With these simplifications the field equations become,

$$\begin{aligned} \nabla \times \mathbf{K} + \mu_0 \frac{\partial \mathbf{P}}{\partial t} &= -\frac{\partial \mathbf{B}_0}{\partial t} \\ \nabla \times \mathbf{P} - \varepsilon_0 \frac{\partial \mathbf{K}}{\partial t} &= 0 \; . \end{aligned} \qquad \text{(B1)}$$

The equations above are valid in vacuum and neglect the earth's di–electric properties. The fields in Eq. (B1) are,

$$\begin{aligned} \mathbf{K} &= \mathbf{E} + \sqrt{\frac{\alpha}{\varepsilon_0}}\, \boldsymbol{\Gamma} \\ \mathbf{P} &= \mathbf{H} + \sqrt{\frac{\beta}{\mu_0}}\, \boldsymbol{\mu} \; , \end{aligned} \qquad \text{(B2)}$$

where

$$
\begin{aligned}
\mathbf{E} \;&= \text{electric field} \\
\mathbf{H} \;&= \text{magnetic field in vacuum} \\
\mathbf{B}_0 \;&= \text{magnetic field inside the core} \\
\mathbf{\Gamma} \;&= \text{induced gravitational field} \\
\boldsymbol{\mu} \;&= \text{mesofield} \\
\varepsilon_0 \;&= \text{vacuum permittivity } (\varepsilon_0 = 8.854 \times 10^{-12} \text{ farad/m}) \\
\mu_0 \;&= \text{vacuum permeability } (\mu_0 = 4\pi \times 10^{-7} \text{ henry/m}) \\
\alpha \;&= \text{permittivity of space to gravity} \\
&\quad (\alpha = 1/4\,\pi\,\gamma = 1.19 \times 10^{9} \text{ s}^2\text{kg/m}^3) \\
\gamma \;&= \text{gravitational constant } (\gamma = 6.67 \times 10^{-11} \text{ m}^3/\text{s}^2\text{kg}) \\
\beta \;&= 1/\alpha c^2 \;\; (\beta = 9.34 \times 10^{-27} \text{ m/kg}) \\
c \;&= \text{velocity of light } (c = 3 \times 10^{8} \text{ m/s}) \;.
\end{aligned}
\tag{B3}
$$

In addition, the fields satisfy the relations (cf. Eqs. (A56a), (A56b), (A56d), and A(56e)),

$$
\begin{aligned}
\nabla \cdot \mathbf{K} &= 0 \\
\nabla \cdot \mathbf{P} &= c\,(\rho_m - \rho_{m_0}) \\
\nabla \cdot \mathbf{B}_0 &= 0 \;.
\end{aligned}
\tag{B4}
$$

In Eq. (B4) $\rho_m - \rho_{m_0}$ is the mass density of the gravitational field produced by the mass m_0 of the craft (Heim 1989). The total field mass, $m - m_0$, is approximately equal to $1.39 \times 10^{-28}\, m_0^2\,(r - r_0)/r\, r_0$ kg if $r > r_0$, where r_0 is the equivalent radius of m_0. The density of such a small field mass can safely be disregarded on the right–hand side of Eq. (B4).

2. *The Hertz Vector*

Equations (B1) and (B4) have the well–known form of Maxwell's equations if $-\partial \mathbf{B}_0/\partial t$ is regarded as a source term in Eq. (B1). Methods of solving the equations have been known for a long time. The procedure described below follows Stratton (1941).

The fields may be expressed in terms of the Hertz vector function $\mathbf{\Pi}$. If $\mathbf{\Pi}$ satisfies the equation

$$
\nabla \times \nabla \times \mathbf{\Pi} \;-\; \nabla\,(\nabla \cdot \mathbf{\Pi}) \;+\; \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{\Pi}}{\partial t^2} \;=\; \frac{1}{\mu_0}\,\mathbf{B}_0 \;, \tag{B5}
$$

then $\mathbf{K}$ and $\mathbf{P}$ are obtained from the pair of relations,

$$\mathbf{K} = -\mu_0 \nabla \times \frac{\partial \mathbf{\Pi}}{\partial t}$$

$$\mathbf{P} = \nabla(\nabla \cdot \mathbf{\Pi}) - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{\Pi}}{\partial t^2} \qquad (\mu_0 \varepsilon_0 = \frac{1}{c^2}) \ . \tag{B6}$$

This can be verified by substituting Eq. (B6) into Eqs. (B1) and (B4) (with $\rho_m - \rho_{m_o} = 0$), making use of Eq. (B5) and the last line of Eq. (B4).

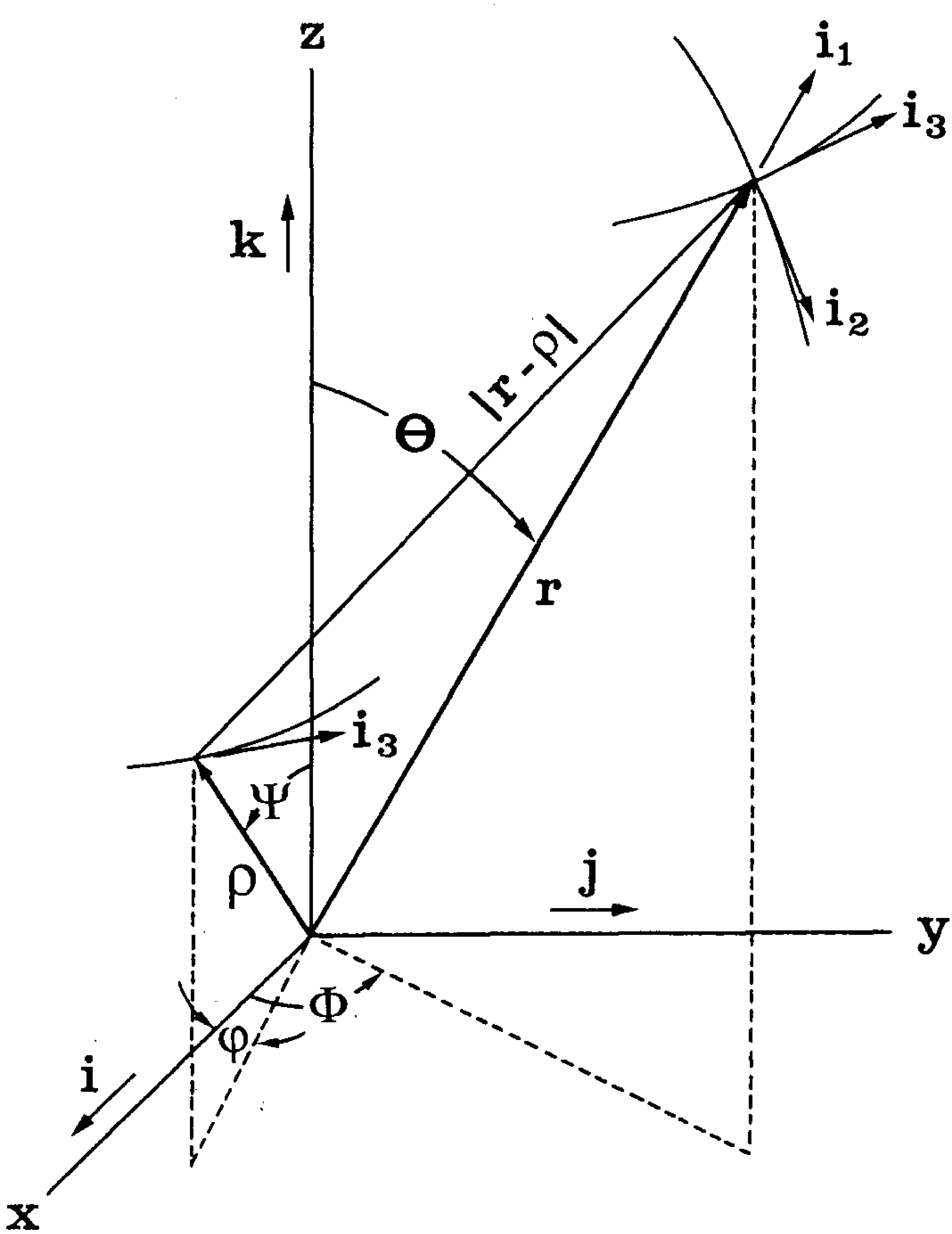

Fig. B1. Coordinate system used for calculating the fields.

In rectangular coordinates the following vector identity is valid:

$$\nabla \times \nabla \times \mathbf{\Pi} = \nabla(\nabla \cdot \mathbf{\Pi}) - \nabla^2 \mathbf{\Pi} \ . \tag{B7}$$

Use of this identity transforms Eq. (B5) into the relation below,

$$\nabla^2 \mathbf{\Pi} - \frac{1}{c^2}\frac{\partial^2 \mathbf{\Pi}}{\partial t^2} = -\frac{1}{\mu_0}\mathbf{B}_0 \ . \tag{B8}$$

Equation (B8) is solved in the coordinate system of Fig. B1. **i**, **j**, and **k** in that figure are unit vectors along the rectangular axes x, y, and z, respectively. Similarly, $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are unit vectors in spherical coordinates, **r** is the radius vector to the point at which $\mathbf{\Pi}$ is calculated, and $\boldsymbol{\rho}$ is the radius vector to a field point of $\mathbf{B}_0(\boldsymbol{\rho},t)$ inside the core. The shape of the latter may be either cylindrical or toroidal according to Fig. 1 of the main text. In either case the field lines of $\mathbf{B}_0$ are circles around the z–axis in a direction tangential to the outer boundary surfaces of the core. A tangential magnetic field **B** is discontinuous across a boundary. In the case of a toroidal magnet the field is zero outside. If the core is cylindrical, as in Fig. 1a, the field outside is reduced by the factor μ_0/μ relative to its value inside (μ = permeability of the core material). Since μ_0/μ is a very small quantity of order 10^{-3}–10^{-5}, $\mathbf{B}_0$ will be assumed to vanish outside the core in the cylindrical case, too.

Neglecting end effects in the case of the cylindrical core the primary magnetic field $\mathbf{B}_0$ in the arrangements of Figs. 1a and 1b is,

$$\begin{aligned} \mathbf{B}_0(\boldsymbol{\rho},t) &= \frac{\mu\, I(t)}{2\pi}\,\frac{\mathbf{j}\cos\varphi - \mathbf{i}\sin\varphi}{\rho\sin\psi} \quad \text{inside the core} \\ &= 0 \quad \text{outside} \ . \end{aligned} \tag{B9}$$

Note that $\mathbf{j}\cos\varphi - \mathbf{i}\sin\varphi = \mathbf{i}_3$ (cf. Fig. B1). The current, I(t), equals the current through a single wire multiplied by the number of wires in a loop of Fig. 1a, or by the number of windings around the toroidal core of Fig. 1b.

All derivations will be carried out for both sine and cosine currents. The subscripts s (sine) and c (cosine) denote the corresponding fields. It is convenient to express $\mathbf{B}_0$ in terms of its two Fourier components,

$$\mathbf{B}_{0s}(\boldsymbol{\rho},t) = \mathbf{B}_0(\boldsymbol{\rho})\frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right)$$

$$\mathbf{B}_{0c}(\boldsymbol{\rho},t) = \mathbf{B}_0(\boldsymbol{\rho})\frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right) \tag{B10}$$

$$\mathbf{B}_0(\boldsymbol{\rho}) = \frac{\mu I_0}{2\pi}\,\frac{\mathbf{j}\cos\varphi - \mathbf{i}\sin\varphi}{\rho\sin\psi} \ ,$$

where I_0 is the maximum number of ampere turns.

A solution of Eq. (B8) is obtained, first of all, for the single Fourier component $\mathbf{B}_0(\boldsymbol{\rho})\exp(-i\omega t)$. From this the expression of $\boldsymbol{\Pi}$ for sine and cosine currents can easily be derived. Keeping only the $\exp(-i\omega t)$ – term in Eq. (B10) a single rectangular component of Π, Π_m, satisfies the e– quation,

$$\nabla^2\Pi_m - \frac{1}{c^2}\frac{\partial^2\Pi_m}{\partial t^2} = -\frac{e^{-i\omega t}}{\mu_0}B_{0m}(\boldsymbol{\rho}) \ . \tag{B11}$$

The solution of Eq. (B11) is

$$\Pi_m = \frac{e^{-i\omega t}}{4\pi\mu_0}\int B_{0m}(\boldsymbol{\rho})\,\frac{e^{ik|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\,d^3\rho$$

$$k = \frac{\omega}{c} \ . \tag{B12}$$

(k should not be confused with the unit vector **k**). The complete vector function now becomes,

$$\boldsymbol{\Pi} = \frac{e^{-i\omega t}}{4\pi\mu_0}\int \mathbf{B}_0(\boldsymbol{\rho})\,\frac{e^{ik|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\,d^3\rho \ . \tag{B13}$$

When $r > \rho$ the function under the integral can be expanded,

$$\frac{e^{ik|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|} = ik\sum_{n=0}^{\infty}(2n+1)\,P_n(\boldsymbol{\Omega}_\rho\cdot\mathbf{i}_1)\,j_n(k\rho)\,h_n^{(1)}(kr) \ , \tag{B14}$$

where $\boldsymbol{\Omega}_\rho$ is a unit vector along $\boldsymbol{\rho}$ and P_n is a Legendre polynomial. The spherical Bessel functions j_n and $h_n^{(1)}$ are given below,

$$j_n(x) = \sqrt{\frac{\pi}{2x}}\,J_{n+1/2}(x)$$

$$h_n^{(1)}(x) = \sqrt{\frac{\pi}{2x}}\,H_{n+1/2}^{(1)}(x) \ . \tag{B15}$$

In the equation above J and H are Bessel and Hankel functions, respectively, of half integral order.

It will be assumed that r is always large compared to the dimensions of the magnet, so that $r \gg \rho$ for all r and ρ. It is then sufficiently accurate to retain only the first two terms in the expansion of Eq. (B14),

$$\frac{e^{ik|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|} \approx ik\Big\{ j_0(k\rho)h_0^{(1)}(kr) + 3\left[\cos\theta\cos\psi + \right.$$

$$\left. + \sin\theta\sin\psi\cos(\Phi-\varphi)\right] j_1(k\rho)\, h_1^{(1)}(kr) \Big\} \; . \tag{B16}$$

Equation (B16) leads to the dipole approximation of **K** and **P**. After inserting Eq. (B16) into Eq. (B13) and expressing $\mathbf{B}_0(\boldsymbol{\rho})$ in terms of Eq. (B10) the Hertz vector becomes,

$$\boldsymbol{\Pi} = ik\frac{I_0}{8\pi^2}\frac{\mu}{\mu_0}e^{-i\omega t}\Big\{ h_0^{(1)}(kr)\iint j_0(k\rho)\rho d\rho d\psi \int_0^{2\pi}(\mathbf{j}\cos\varphi - \mathbf{i}\sin\varphi)d\varphi +$$

$$+ 3h_1^{(1)}(kr)\iint j_1(k\rho)\rho d\rho d\psi \int_0^{2\pi}[\cos\theta\cos\psi +$$

$$+ \sin\theta\sin\psi\cos(\Phi-\varphi)]\,(\mathbf{j}\cos\varphi - \mathbf{i}\sin\varphi)d\varphi \Big\} \; . \tag{B17}$$

The first line of Eq. (B17) gives no contribution to $\boldsymbol{\Pi}$ because the integral over φ is zero. In the second line the $\cos\theta\cos\psi$ – term vanishes for the same reason. In the remaining integral $j_1(k\rho)$ may be approximated by

$$j_1(k\rho) \approx \frac{1}{3}k\rho \; , \tag{B18}$$

because $k\rho$ is, in general, a very small quantity. This reduces the Hertz vector of Eq. (B17) to the expression,

$$\boldsymbol{\Pi} = ik^2\frac{I_0}{8\pi^2}\frac{\mu}{\mu_0}e^{-i\omega t}\, h_1^{(1)}(kr)\sin\theta\iint\rho^2 d\rho\sin\psi d\psi \times$$

$$\times\int_0^{2\pi}\cos(\Phi-\varphi)(\mathbf{j}\cos\varphi - \mathbf{i}\sin\varphi)d\varphi$$

$$= ik^2\frac{I_0}{16\pi^2}\frac{\mu}{\mu_0}e^{-i\omega t}\, h_1^{(1)}(kr)\sin\theta\,\mathbf{i}_3\cdot 2\pi\iint\rho^2 d\rho\sin\psi d\psi \; . \tag{B19}$$

The double integral in Eq. (B19), multiplied by 2π, is just the volume V of the core, irrespective of its shape. Hence, the final result,

$$\mathbf{\Pi} = ik^2 \frac{I_0 V}{16\pi^2} \frac{\mu}{\mu_0} e^{-i\omega t} h_1^{(1)}(kr) \sin\theta \, \mathbf{i}_3 \ , \tag{B20}$$

applies to both configurations of Fig. 1. The direction of $\mathbf{\Pi}$ is parallel to $\mathbf{B}_0$.

The expressions for $\mathbf{\Pi}_s$ and $\mathbf{\Pi}_c$ are now obtained by repeating the procedure for the second Fourier component, $\mathbf{B}_0(\rho)\exp(i\omega t)$, or by taking complex conjugates, and combining results according to Eq. (B10). The $\mathbf{\Pi}$–vectors for sine and cosine currents then become,

$$\begin{aligned} \mathbf{\Pi}_s &= \mathbf{i}_3 \frac{I_0 V}{16\pi^2} \frac{\mu}{\mu_0} \left[\frac{1}{r^2} \sin(\omega t - kr) + \frac{k}{r} \cos(\omega t - kr)\right] \sin\theta \\ \mathbf{\Pi}_c &= \mathbf{i}_3 \frac{I_0 V}{16\pi^2} \frac{\mu}{\mu_0} \left[\frac{1}{r^2} \cos(\omega t - kr) - \frac{k}{r} \sin(\omega t - kr)\right] \sin\theta \ . \end{aligned} \tag{B21}$$

3. *The Fields Generated by a Stationary Magnet*

The field vectors $\mathbf{K}$ and $\mathbf{P}$ are derived from Eq. (B6). The curl in the first line of that equation, applied to a function $\mathbf{i}_3 F_3$ in spherical coordinates, is

$$\nabla \times \mathbf{i}_3 F_3 = \mathbf{i}_1 \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta} (\sin\theta \, F_3) - \mathbf{i}_2 \frac{1}{r} \frac{\partial}{\partial r} (r F_3) \ . \tag{B22}$$

The divergence, $\nabla \cdot \mathbf{\Pi}$, in the second line of Eq. (B6) is zero because $\mathbf{\Pi}$ is independent of azimuthal angle Φ. The result of substituting the Hertz vectors of Eq. (B21) into Eq. (B6) is

$$\begin{aligned} \mathbf{K}_s = -\frac{\mu_0 I_0 V\omega}{16\pi^2} \frac{\mu}{\mu_0} \Big\{ & \frac{1}{r^3} (2\mathbf{i}_1 \cos\theta + \mathbf{i}_2 \sin\theta) \cos(\omega t - kr) - \\ & - \frac{k}{r^2} (2\mathbf{i}_1 \cos\theta + \mathbf{i}_2 \sin\theta) \sin(\omega t - kr) - \mathbf{i}_2 \frac{k^2}{r} \sin\theta \cos(\omega t - kr) \Big\} \\ \\ \mathbf{K}_c = \frac{\mu_0 I_0 V\omega}{16\pi^2} \frac{\mu}{\mu_0} \Big\{ & \frac{1}{r^3} (2\mathbf{i}_1 \cos\theta + \mathbf{i}_2 \sin\theta) \sin(\omega t - kr) + \\ & + \frac{k}{r^2} (2\mathbf{i}_1 \cos\theta + \mathbf{i}_2 \sin\theta) \cos(\omega t - kr) - \mathbf{i}_2 \frac{k^2}{r} \sin\theta \sin(\omega t - kr) \Big\} \end{aligned} \tag{B23}$$

$$\mathbf{P}_s = \mathbf{i}_3 k^2 \frac{I_0 V}{16\pi^2} \frac{\mu}{\mu_0} \left\{ \frac{1}{r^2} \sin(\omega t - kr) + \frac{k}{r} \cos(\omega t - kr) \right\} \sin\theta$$

$$\mathbf{P}_c = \mathbf{i}_3 k^2 \frac{I_0 V}{16\pi^2} \frac{\mu}{\mu_0} \left\{ \frac{1}{r^2} \cos(\omega t - kr) - \frac{k}{r} \sin(\omega t - kr) \right\} \sin\theta \ . \tag{B24}$$

These equations show that **K** and **P** are perpendicular to each other.

4. *Coordinate Transformations*

The effect of counterrotating magnets and the integral over the earth are best evaluated in rectangular coordinates. For this purpose the unit vectors in spherical coordinates, $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are expressed in terms of **i**, **j**, and **k**. The transformation formulas and the inverse transformations are,

$$\begin{aligned} \mathbf{i}_1 &= \mathbf{i}\sin\theta\cos\Phi + \mathbf{j}\sin\theta\sin\Phi + \mathbf{k}\cos\theta \\ \mathbf{i}_2 &= \mathbf{i}\cos\theta\cos\Phi + \mathbf{j}\cos\theta\sin\Phi - \mathbf{k}\sin\theta \\ \mathbf{i}_3 &= -\mathbf{i}\sin\Phi + \mathbf{j}\cos\Phi \end{aligned} \tag{B25}$$

$$\begin{aligned} \mathbf{i} &= \mathbf{i}_1\sin\theta\cos\Phi + \mathbf{i}_2\cos\theta\cos\Phi - \mathbf{i}_3\sin\Phi \\ \mathbf{j} &= \mathbf{i}_1\sin\theta\sin\Phi + \mathbf{i}_2\cos\theta\sin\Phi + \mathbf{i}_3\cos\Phi \\ \mathbf{k} &= \mathbf{i}_1\cos\theta - \mathbf{i}_2\sin\theta \ . \end{aligned} \tag{B26}$$

Transforming the unit vectors in Eqs. (B23) and (B24) by means of Eq. (B25) leads to the following expressions for the fields:

$$\begin{aligned} \mathbf{K}_s = -\frac{\mu_0 I_0 V\omega}{16\pi^2} \frac{\mu}{\mu_0} \Big\{ & \frac{1}{r^3} \Big[3\mathbf{i}\sin\theta\cos\theta\cos\Phi + 3\mathbf{j}\sin\theta\cos\theta\sin\Phi + \\ & + \mathbf{k}(3\cos^2\theta - 1) \Big] \cos(\omega t - kr) - \frac{k}{r^2} \Big[3\mathbf{i}\sin\theta\cos\theta\cos\Phi + \\ & + 3\mathbf{j}\sin\theta\cos\theta\sin\Phi + \mathbf{k}(3\cos^2\theta - 1) \Big] \sin(\omega t - kr) - \\ & - \frac{k^2}{r} \Big[\mathbf{i}\sin\theta\cos\theta\cos\Phi + \mathbf{j}\sin\theta\cos\theta\sin\Phi - \mathbf{k}\sin^2\theta \Big] \cos(\omega t - kr) \Big\} \end{aligned}$$

(B27)

$$\mathbf{K}_c = \frac{\mu_0 I_0 V\omega}{16\pi^2}\frac{\mu}{\mu_0}\Big\{\frac{1}{r^3}\Big[3\mathbf{i}\sin\theta\cos\theta\cos\Phi + 3\mathbf{j}\sin\theta\cos\theta\sin\Phi + \\ + \mathbf{k}(3\cos^2\theta - 1)\Big]\sin(\omega t - kr) + \frac{k}{r^2}\Big[3\mathbf{i}\sin\theta\cos\theta\cos\Phi + \\ + 3\mathbf{j}\sin\theta\cos\theta\sin\Phi + \mathbf{k}(3\cos^2\theta - 1)\Big]\cos(\omega t - kr) - \\ - \frac{k^2}{r}\Big[\mathbf{i}\sin\theta\cos\theta\cos\Phi + \mathbf{j}\sin\theta\cos\theta\sin\Phi - \mathbf{k}\sin^2\theta\Big]\sin(\omega t - kr)\Big\}$$

$$\mathbf{P}_s = k^2\frac{I_0 V}{16\pi^2}\frac{\mu}{\mu_0}\Big\{\frac{1}{r^2}\sin(\omega t - kr) + \frac{k}{r}\cos(\omega t - kr)\Big\}(-\mathbf{i}\sin\theta\sin\Phi + \mathbf{j}\sin\theta\cos\Phi)$$

(B28)

$$\mathbf{P}_c = k^2\frac{I_0 V}{16\pi^2}\frac{\mu}{\mu_0}\Big\{\frac{1}{r^2}\cos(\omega t - kr) - \frac{k}{r}\sin(\omega t - kr)\Big\}(-\mathbf{i}\sin\theta\sin\Phi + \mathbf{j}\sin\theta\cos\Phi)$$

Equation (10) of the main text is obtained by solving Eq. (B2) for $\mathbf{\Gamma}$, putting $\mathbf{E} = 0$, replacing $\mathbf{K}$ by $\mathbf{K}_s$ of Eq. (B27), and keeping only the vertical components of $\mathbf{K}_s$. The vertical components are the coefficients of the unit vector $\mathbf{k}$.

Making use of the relations,

$$x = r\sin\theta\cos\Phi, \quad y = r\sin\theta\sin\Phi, \quad z = r\cos\theta, \quad r = \sqrt{x^2 + y^2 + z^2} \qquad \text{(B29)}$$

allows the fields to be fully expressed in rectangular coordinates,

$$\mathbf{K}_s = -\frac{\mu_0 I_0 V\omega}{16\pi^2}\frac{\mu}{\mu_0}\Big\{\frac{1}{r^5}\Big[3\mathbf{i}xz + 3\mathbf{j}yz + \mathbf{k}(3z^2 - r^2)\Big]\cos(\omega t - kr) - \\ - \frac{k}{r^4}\Big[3\mathbf{i}xz + 3\mathbf{j}yz + \mathbf{k}(3z^2 - r^2)\Big]\sin(\omega t - kr) - \\ - \frac{k^2}{r^3}\Big[\mathbf{i}xz + \mathbf{j}yz + \mathbf{k}(z^2 - r^2)\Big]\cos(\omega t - kr)\Big\} \qquad \text{(B30a)}$$

$$\mathbf{K}_c = \frac{\mu_0 I_0 V\omega}{16\pi^2}\frac{\mu}{\mu_0}\Big\{\frac{1}{r^5}\Big[3\mathbf{i}xz + 3\mathbf{j}yz + \mathbf{k}(3z^2 - r^2)\Big]\sin(\omega t - kr) + \\ + \frac{k}{r^4}\Big[3\mathbf{i}xz + 3\mathbf{j}yz + \mathbf{k}(3z^2 - r^2)\Big]\cos(\omega t - kr) - \\ - \frac{k^2}{r^3}\Big[\mathbf{i}xz + \mathbf{j}yz + \mathbf{k}(z^2 - r^2)\Big]\sin(\omega t - kr)\Big\} \qquad \text{(B30b)}$$

$$\mathbf{P}_s = k^2 \frac{I_0 V}{16\pi^2} \frac{\mu}{\mu_0} \left\{ \frac{1}{r^3} \sin(\omega t - kr) + \frac{k}{r^2} \cos(\omega t - kr) \right\} (-\mathbf{i}y + \mathbf{j}x) \qquad \text{(B31a)}$$

$$\mathbf{P}_c = k^2 \frac{I_0 V}{16\pi^2} \frac{\mu}{\mu_0} \left\{ \frac{1}{r^3} \cos(\omega t - kr) - \frac{k}{r^2} \sin(\omega t - kr) \right\} (-\mathbf{i}y + \mathbf{j}x) \ . \qquad \text{(B31b)}$$

5. *The Rotating Fields*

The time dependence of the vertical field components is largely eliminated by employing two counterrotating magnets and adding their fields together. In order to determine the effect of rotation on the fields as seen by a stationary observer the fields of Eqs. (B30) and (B31) are attached to two coordinate systems rotating about the y–axis in opposite directions. Figure B2 shows the stationary and rotating coordinate systems of magnets 1 and 2. The angular frequency of rotation, ω, is the same for both and is also equal to the current frequency. Certain phase relations must be observed between all components of the system. The phases are so chosen that most terms in the vertical component of $\mathbf{K} = \mathbf{K}_s + \mathbf{K}_c$ become stationary.

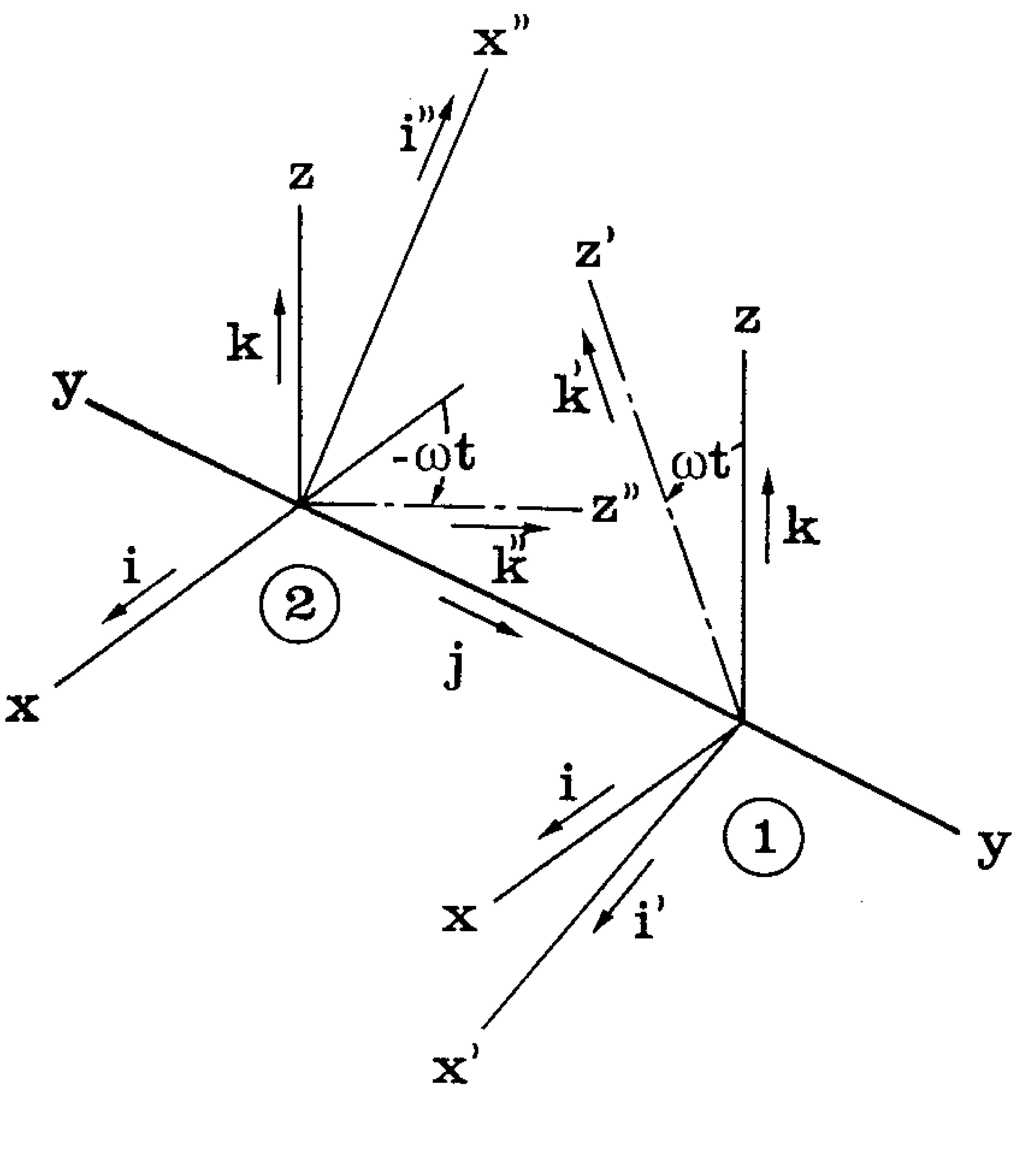

Fig. B2. Stationary and counterrotating coordinates.

The sense of rotation shown in Fig. B2 is anticlockwise in (1) and clockwise in (2). The first magnet rotates as $\cos\omega t$, the second as $\cos(-\omega t - \pi/2) = -\sin\omega t$. The coordinate transformations corresponding to these rotations are

1st Magnet

$$\begin{aligned} x' &= x\cos\omega t - z\sin\omega t & \mathbf{i}' &= \mathbf{i}\cos\omega t - \mathbf{k}\sin\omega t \\ y' &= y & \mathbf{j}' &= \mathbf{j} \\ z' &= x\sin\omega t + z\cos\omega t & \mathbf{k}' &= \mathbf{i}\sin\omega t + \mathbf{k}\cos\omega t \end{aligned} \tag{B32}$$

2nd Magnet

$$\begin{aligned} x'' &= -x\sin\omega t + z\cos\omega t & \mathbf{i}'' &= -\mathbf{i}\sin\omega t + \mathbf{k}\cos\omega t \\ y'' &= y & \mathbf{j}'' &= \mathbf{j} \\ z'' &= -x\cos\omega t - z\sin\omega t & \mathbf{k}'' &= -\mathbf{i}\cos\omega t - \mathbf{k}\sin\omega t \ . \end{aligned} \tag{B33}$$

The distance r remains invariant under the transformations above,

$$x'^2 + y'^2 + z'^2 = x''^2 + y''^2 + z''^2 = x^2 + y^2 + z^2 = r^2 \ . \tag{B34}$$

In addition, the currents vary as $\sin\omega t$ through the first magnet and as $\cos\omega t$ through the second.

All coordinates and unit vectors in $\mathbf{K}_s$, Eq. (B30a), and $\mathbf{P}_s$, Eq. (B31a), are now replaced by primed quantities. Likewise, all coordinates and unit vectors of $\mathbf{K}_c$ and $\mathbf{P}_c$ in Eqs. (B30b) and (B31b) are replaced by doubly primed symbols. In this way the fields are made to rotate with the singly and doubly primed coordinate axes. The fields observed in the stationary system are obtained by replacing all singly and doubly primed quantities by their respective expressions on the right–hand sides of Eqs. (B32) and (B33). Finally, sine and cosine fields are added vectorially to give,

$$\mathbf{K} = \mathbf{K}_s + \mathbf{K}_c$$
$$\mathbf{P} = \mathbf{P}_s + \mathbf{P}_c \ . \tag{B35}$$

The final result of these operations is

$$\begin{aligned}\mathbf{K} = -\frac{\mu_0 I_0 V\omega}{16\pi^2}\frac{\mu}{\mu_0}\Big\{ &\frac{1}{r^5}\Big(\mathbf{i}\left[3xz\cos kr + (3x^2 - r^2)\sin(2\omega t - kr)\right] + \\ &+ 3\mathbf{j}y\left[x\sin(2\omega t - kr) + z\cos kr\right] + \mathbf{k}\left[3xz\sin(2\omega t - kr) + \right. \\ &\left. + (3z^2 - r^2)\cos kr\right]\Big) + \frac{k}{r^4}\Big(\mathbf{i}\left[3xz\sin kr + (3x^2 - r^2)\cos(2\omega t - kr)\right] + \\ &+ 3\mathbf{j}y\left[x\cos(2\omega t - kr) + z\sin kr\right] + \mathbf{k}\left[3xz\cos(2\omega t - kr) + \right. \\ &\left. + (3z^2 - r^2)\sin kr\right]\Big) - \frac{k^2}{r^3}\Big(\mathbf{i}\left[xz\cos kr + (x^2 - r^2)\sin(2\omega t - kr)\right] + \\ &+ \mathbf{j}y\left[x\sin(2\omega t - kr) + z\cos kr\right] + \\ &+ \mathbf{k}\left[xz\sin(2\omega t - kr) + (z^2 - r^2)\cos kr\right]\Big)\Big\}\end{aligned} \tag{B36}$$

$$\begin{aligned}\mathbf{P} = k^2\frac{I_0 V}{16\pi^2}\frac{\mu}{\mu_0}\Big\{ &\frac{1}{r^3}\Big(\mathbf{i}y\sin kr - \mathbf{j}\left[x\sin kr - z\cos(2\omega t - kr)\right] - \\ &- \mathbf{k}y\cos(2\omega t - kr)\Big) + \frac{k}{r^2}\Big(-\mathbf{i}y\cos kr + \mathbf{j}\left[x\cos kr - \right. \\ &\left. - z\sin(2\omega t - kr)\right] + \mathbf{k}y\sin(2\omega t - kr)\Big)\Big\} \ .\end{aligned} \tag{B37}$$

The field lines of **K** in the y–z–plane are plotted to scale in Fig. 8 for a wave number $k = 2.73 \times 10^{-7}\ \mathrm{m}^{-1}$ and somewhat schematically for $k = 1.57 \times 10^{-6}\ \mathrm{m}^{-1}$ in Fig. 9. Circular field lines, separating antigravitational from gravitational zones, occur whenever the condition

$$1 + kr\tan kr = 0 \tag{B38}$$

is satisfied. The first 5 values of kr meeting this requirement are

$$kr = 2.798386,\ 6.121250,\ 9.317866,\ 12.486454,\ 15.644128\ .$$

The separation between $(kr)_{n+1}$ and $(kr)_n$ approaches π with increasing n.

The electric, magnetic, and gravitational fields, including the mesofield, are obtained from Eqs. (B36) and (B37) by use of Eq. (B2),

$$\begin{aligned} \mathbf{E} &= \mathbf{K} \\ \mathbf{H} &= \mathbf{P} \qquad (\mathbf{B} = \mu_0 \mathbf{H}) \\ \boldsymbol{\Gamma} &= \sqrt{\frac{\varepsilon_0}{\alpha}}\,\mathbf{K} \\ \boldsymbol{\mu} &= \sqrt{\frac{\mu_0}{\beta}}\,\mathbf{P}\ . \end{aligned} \tag{B39}$$

The mesofield $\boldsymbol{\mu}$ acts only on moving masses, on which it exerts a Lorentz–type force (cf. Eq. (A56g)),

$$\mathbf{F}_\mu = \beta m\ \mathbf{v} \times \boldsymbol{\mu}\ , \tag{B40}$$

where $\mathbf{v}$ is the velocity of mass m and β is given in Eq. (B3).

The vertical component of the gravitational field, Γ_z, is derived directly from Eqs. (B39) and (B36). Transforming x, y, and z back into spherical coordinates by means of Eq. (B29) Γ_z becomes,

$$\begin{aligned} \Gamma_z = -\sqrt{\frac{\varepsilon_0}{\alpha}}\,\frac{\mu_0 I_0 V\omega}{16\pi^2}\,\frac{\mu}{\mu_0}\Big\{ & \frac{1}{r^3}\Big[3\sin\theta\cos\theta\cos\Phi\sin(2\omega t - kr) + \\ & + (3\cos^2\theta - 1)\cos kr\Big] + \frac{k}{r^2}\Big[3\sin\theta\ \cos\theta\cos\Phi\cos(2\omega t - kr) + \\ & + (3\cos^2\theta - 1)\sin kr\Big] - \frac{k^2}{r}\Big[\sin\theta\cos\theta\cos\Phi\sin(2\omega t - kr) - \\ & - \sin^2\theta\sin kr\Big]\Big\}\ . \end{aligned} \tag{B41}$$

As this equation shows, the vertical field is not independent of time. However, the time–dependent terms do not contribute to the integral over the earth because $\cos\Phi$, integrated from 0 to 2π, gives zero. On the other hand, the factors $(3x^2 - r^2)$, occurring in the x–component of $\mathbf{K}$, do not integrate to zero. The x–component of $\boldsymbol{\Gamma}$ containing these terms will be denoted by Γ_x. It provides an oscillatory gravitational force in a direction perpendicular to the axis of rotation and is given by the expression,

$$\Gamma_x = -\sqrt{\frac{\varepsilon_0}{\alpha}}\frac{\mu_0 I_0 V\omega}{16\pi^2}\frac{\mu}{\mu_0}\left\{\frac{1}{r^3}(3\sin^2\theta\cos^2\Phi - 1)\sin(2\omega t - kr) + \right.$$
$$+ \frac{k}{r^2}(3\sin^2\theta\cos^2\Phi - 1)\cos(2\omega t - kr) -$$
$$\left. - \frac{k^2}{r}(\sin^2\theta\cos^2\Phi - 1)\sin(2\omega t - kr)\right\} . \qquad \text{(B42)}$$

It should be possible to eliminate this sideway oscillation by mounting a second pair of counterrotating magnets turned through 180° relative to the first.

A strong electric field **E** is induced in addition to the relatively weak gravitational field. **E**, according to Eq. (B39), is given by Eq. (B36) and contains both stationary terms and terms oscillating with an angular frequency 2ω. The electric field gives rise to the phenomena mentioned in the Conclusions of the main text.

Of particular interest from a theoretical point of view is the observation of dark rings surrounding a UFO, as seen by W. A. Webb through his polaroid sunglasses (Roush 1968). Assuming the rings to be caused by the Faraday–rotation of the plane of polarized light due to the Zeeman effect, W. K. Allan has estimated the magnetic field strength required to explain the observation. His calculations are reported by Ch. A. Maney (1965).

According to the present theory, the antigravitational field is accompanied by a strong electric field and not by a strong magnetic field. However, the same rotation of the plane of polarization may be achieved by an electric field via the Stark effect. This phenomenon is known as the Kerr effect. Since the basic form of the electric field is known from Eq. (B36) the Kerr effect can be calculated exactly. This would yield valuable information about the electric field strength, and, by use of Eq. (B39), about the strength of Γ.

6. *The Antigravitational Force*

If ρ_m is the average mass density of the earth, then the gravitational force **F** acting on the magnets is equal to the following integral of Γ_z, extended over the volume of the earth:

$$\mathbf{F} = -\mathbf{k}\rho_m \int_h^{D+h} r^2 dr \int_{(\cos\theta)_{min}}^{1} d(\cos\theta) \int_0^{2\pi} \Gamma_z(r, -\theta, \Phi, t)\, d\Phi \ , \qquad \text{(B43)}$$

where

$$D = \text{diameter of the earth } (D = 1.2757 \times 10^7 \text{ m})$$

$$h = \text{height of magnet above the earth's surface}$$

$$(\cos\theta)_{min} = \frac{h(D+h)+r^2}{2(R+h)r} \; .$$

Γ_z in this equation is taken from Eq. (B41). As mentioned above, the time–dependent terms in Γ_z do not contribute to the integral. Performing the integration of Eq. (B43) results in the expression,

$$F = -k\frac{c\rho_m}{8}\sqrt{\frac{\varepsilon_0}{\alpha}}\,\frac{\mu_0 I_0 V}{2\pi}\,\frac{\mu}{\mu_0}\,g(k,h) \tag{B44}$$

$$\begin{aligned} g(k,h) = \frac{1}{(R+h)^3}\Big\{ & -D(D+2h)\big[\sin kh + \sin k(D+h)\big] + \\ & + \frac{4}{k}\big[h\cos kh - (D+h)\cos k(D+h)\big] - \\ & - \frac{4}{k^2}\big[\sin kh - \sin k(D+h)\big]\Big\} \; . \end{aligned}$$

The g(k,h)–function is plotted in Figs. 10 and 11 of the main text.

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